Direction des Études et Synthèses Économiques

G 2015 / 01

The walking dead Euler equation
Addressing a challenge to monetary policy models

Aurélien POISSONNIER

Document de travail

Institut National de la Statistique et des Études Économiques
The walking dead Euler equation
Addressing a challenge to monetary policy models

Aurélien POISSONNIER*

JANVIER 2015


* Département des Études Économiques - Timbre « Études Macroeconomiques» Timbre G220 - 15, bd Gabriel Péri - BP 100 - 92244 MALAKOFF CEDEX
Crest-LMA & École Polytechnique
The walking dead Euler equation
Addressing a challenge to monetary policy models

Abstract

Despite some strong cases built against it, the Euler equation on consumption remains a cornerstone of monetary policy models.

In this paper I test the representative household's consumption-savings trade-off in two original directions. I first use households' specific interest rates for both US and France. These rates have a better explanatory power of the representative consumer's behaviour than the monetary policy rate. I also use a less restrictive approach to measure households' expectations based on survey data.

However, the challenge posed by the Euler equation to monetary policy models remains.

Keywords: Euler equation, consumption

L'équation d'Euler est morte, vive l'équation d'Euler !

Résumé

Malgré des éléments à charge conséquents, l’équation d’Euler sur la consommation reste une des briques élémentaires des modèles de politique monétaire.


Malgré cela, le challenge posé par l’équation d’Euler aux modèles de politique monétaire reste entier.

Mots-clés : équation d’Euler, consommation

Classification JEL : E21, D91
Introduction

The Euler equation on consumption is a cornerstone of monetary policy models. In the permanent income hypothesis, when households are assumed to be rational, their consumption scheme should verify the following equation:

\[ 1 = \beta E_t \left( \frac{U_c(t+1)}{U_c(t)} \right) \frac{1 + i_t}{\Pi_{t+1}} \]  \hspace{1cm} (1)

where \( U_c(t) \) is the marginal utility of consumption at time \( t \), \( i_t \) is the nominal interest rate at which households can lend and borrow and \( \Pi_t \) the inflation rate of consumption prices. \( E_t \) is the expectation operator describing how households form their expectations over future variables at time \( t \).

Since Hall (1978), macroeconomic data have been confronted to economic theory repeatedly leading to successive refutations, validations and improvements in households’ consumption models. Early papers, surveyed by Attanasio (1999), have investigated the properties of consumption within this framework under the assumption of constant real interest rate and often certainty equivalence. Many alternative utility functions have been used to reconcile consumption data with the permanent-income hypothesis and understand some major stylized facts. Mankiw (1982), and Bernanke (1985) study the specificities of durables consumption. Abel (1990) and Gali (1994) investigate the equity premium puzzle by modifying the utility function. Campbell and Mankiw (1990) consider a population with only a fraction of households following the model, while others, financially constrained, consume their current income in each period. Flavin (1981) investigates the excess sensitivity puzzle, that is consumption reacting to lagged changes in income. Campbell and Deaton (1989) investigate the excess smoothness puzzle, that is consumption not responding one-to-one to shocks to permanent income.

Relaxing the constant interest rate hypothesis Canzoneri et al. (2007) test the first order condition of households maximization program (1) under different utility specifications against the data. Their main result is that when the Euler equation is assumed to hold and the interest rate is treated as the unknown variable, the counter-factual interest rate to which the representative household seems to respond is quite different from the monetary policy rate. In most cases it is negatively correlated to it, a result they pose as a challenge for monetary policy models, which are built on this Euler equation to link the real and nominal side of the economy.

To derive these results, they assume that households’ expectations can be described through a VARX model, i.e. households use optimally all the information publicly available. This hypothesis combined either with a log-normality hypothesis or a first order approximation allows them to estimate the marginal rate of substitution between present and future consumption which, if the Euler equation was verified by the data, should be equal to the interest rate faced by households. They compare this marginal rate of substitution with the actual monetary policy interest rate and doing so invalidate the permanent income hypothesis even when many alternative utility functions are considered.

Carroll (2000) would interpret this result as an incompatibility between macro data and a microfounded model, due to the differences in the marginal behaviour of individuals. The Euler
equation on consumption could be wrong, not because of the form of the utility function but because the representative agent assumption is heroic. However, microeconomics comes against its own difficulties (Carroll, 2001; Ludvigson and Paxson, 2001) while macroeconomic models remain widespread and have crucial implications for the policy maker, both monetary and fiscal. Thus, the challenge posed by the aggregate Euler equation on consumption is worth investigating.

I built on Canzoneri et al.'s analysis and test household specific rates in an estimated Euler equation for both the US and France when expectations are measured ex ante by a VARX model. I find that households' behaviour is better described by interest rates other than the monetary policy one; indicating that households encounter sizeable frictions on the loans and savings market.

Specifically, I find that the Euler equation is compatible with households reacting to the rate on car loans in the US; but for France households do not seem to react to any specific rates. These results have strong implications for the conduct of monetary policy: the transmission channel is much weaker than generally assumed; and for economic modelling: the spread between household market conditions and the monetary policy instrument can not be relegated to the residual.

Even for the US, there remain sizeable discrepancies between the Euler equation and the data. In particular households seem to react to an interest rate much more volatile than actual rates, a property I will investigate further in the remainder of this project.
1 Testing the Euler equation against household specific interest rates

Using macroeconomic data to capture the value of structural parameters is not the purpose of this paper: Attanasio and Weber (1993) show that estimates of the intertemporal elasticity of substitution on aggregate data are systematically lower than estimates on average cohort data. Attanasio and Weber (1993, 1995) show that tests on aggregate data tend to reject micro-founded models specification too often. Carroll (2000) actually wrote a requiem for the representative consumer model on the idea that individual wealth heterogeneity imply marginal utility heterogeneity and potentially impossible aggregation of the Euler equation. However as this model still prevails in macroeconomics, I inquire into how much of the Euler equation can be saved by a richer dataset.

Canzoneri et al. (2007) test and reject the Euler equation on consumption on US data.¹ They describe the failure of Euler equation as follows: "the consumption Euler equation implies that the real interest rate is proportional to the expected growth of real consumption. The empirical literature shows that a monetary tightening has a small effect on consumption in the first quarter following the tightening. In the following few quarters, the consumption falls more rapidly so that expected consumption growth declines. A decline in expected consumption growth will reduce the real interest rate implied by the Euler equation. The empirical literature shows that money market rates respond in the opposite direction."

However, if households were reacting to another interest rate than that of the money market (mortgage, deposit, private loans...), the difference between this rate and monetary policy could account for the negative or weak correlation they observe. Such a result would have strong implications for monetary policy analysis and the way it is modelled (in the New Neoclassical Synthesis for instance). It would rehabilitate the Euler equation but dampen the transmission mechanism between the policy maker and households.

In this paper, I investigate this possibility. I estimate the relation between different household specific interest rates, expected consumption growth and expected inflation and assess the significance of interest rates in this equation and the fit to the data. The interest rates I consider are depicted on Figure 1. The correlation of real monetary policy rate and the spreads on household specific rates are reported in Table 1. It is noteworthy that household specific spreads are negatively correlated to real monetary policy rate, which could a priori account for the results exposed by Canzoneri et al. (2007).

US rates (Figure 1a)

¹In appendix C I replicate their results. I show that they also hold on French data. While they tested the robustness to the definition of the utility function considered, I show that the result is also robust to the choice of several consumption bundles.
Table 1: Correlation of the interest rates (in real terms)

<table>
<thead>
<tr>
<th>rate</th>
<th>real Fedfunds</th>
<th>spread Mortgage</th>
<th>spread Car Loans</th>
<th>spread Pers. Loans</th>
<th>spread Cert. Deposit</th>
<th>spread Treasury Bill</th>
<th>real Pibor</th>
<th>spread Deposit</th>
<th>spread Livret</th>
<th>spread Mortgage</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) US</td>
<td>1</td>
<td>-0.3</td>
<td>0.43</td>
<td>-0.46</td>
<td>-0.2</td>
<td>-0.46</td>
<td>1</td>
<td>-0.5</td>
<td>-0.83</td>
<td>-0.11</td>
</tr>
<tr>
<td>(b) France</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

on Personal Loans at Commercial Banks, 24 Month Loan), a deposit rate (3-Month Certificate of Deposit: Secondary Market Rate) and the 3-month treasury bill (3-Month Treasury Bill: Secondary Market Rate). Apart from the mortgage rate, maturities of these interest rates are short. The mortgage rate is included because wealth effects are empirically stronger in the US than in France, a stylized fact sometimes attributed to the higher flexibility of mortgages in the US where real estate can be used as a collateral for consumption (Aviat et al., 2007). Some of these rates are closely related (treasury bill, fedfunds and certificates of deposits). I keep them all to remain as exhaustive as possible.²

French rates (Figure 1b) I use the 3-month interbank rate (Pibor-Euribor) and three different rates, all for new contracts for households and individual enterprises: Deposits with agreed maturity up to one year, Deposits redeemable at notice and Loans for house purchases excl. bank overdrafts, total maturity. Deposits redeemable at notice refer to regulated savings accounts. The oldest one (livret A) dates back to the nineteenth century and is the most widespread financial product in France; more than 90% of French residents have such an account, standing with other livrets (livret jeune, livret developement durable, livret d’épargne populaire) for more than 15% of French households’ financial assets (Noyer and Mérieux, 2012). As shown on Figure 1b the return on these savings is not linked to the monetary policy rate on most of the sample. More recently, this rate has been indexed on a combination of inflation, Eonia and Euribor with a discretionary component set by the government.²

² Data are further detailed in appendix A.1.
Figure 1: Actual interest rates in nominal terms
The Euler equation under a linear form I assume that households are forming their expectations through a VARX model estimated ex-ante.

\[ c_{t+1} = \sum_{j=0}^{k} \lambda_{j}^{c,c} c_{t-j} + \sum_{j=0}^{k} \lambda_{j}^{c,\pi} P_{t-j} + \gamma^{c} X_{t} + \epsilon_{t+1}^{c} \]  
\[ P_{t+1} = \sum_{j=0}^{k} \lambda_{j}^{\pi,c} c_{t-j} + \sum_{j=0}^{k} \lambda_{j}^{\pi,\pi} P_{t-j} + \gamma^{\pi} X_{t} + \epsilon_{t+1}^{\pi} \]  

with \( c \) consumption per capita (in log) and \( P \) its deflator (in log) regressed on 12 lags of both variables and on exogenous regressors (\( X \)). For these regressors I consider alternatively economic variables, following Canzoneri et al. (2007) and Fuhrer (2000), and balances of opinion from household surveys. In this latter case, I use the surveys of consumers by the university of Michigan and the European harmonized household survey published for France by Insee (see also Appendix A.2). This second method is less biased by the economist’s \( a \) \( pri \) \( ori \), however I find very similar results with these data. \( \epsilon^{c}, \epsilon^{\pi} \) are the errors of this VARX model and are by assumption identified to forecasting errors.

If the Euler equation holds, the following should hold under log-normality hypothesis of the VARX residuals (see Appendix B):

\[ i_{t} = \sigma E_{t} \Delta c_{t+1} + E_{t} \pi_{t+1} - \ln(\beta) - \left( \frac{\sigma^{2}}{2} \text{var}(\epsilon^{c}) + \sigma \text{cov}(\epsilon^{c}, \epsilon^{\pi}) + \frac{1}{2} \text{var}(\epsilon^{\pi}) \right) \]

with \( i_{t} \) the interest rate to which households are subject and \( E_{t} \Delta c_{t+1}, E_{t} \pi_{t+1} \) the expectations formed through the previous VARX model for consumption growth (\( \Delta \log \)) and inflation.\(^3\)

Model estimation Using several interest rates in an agnostic way, the model can be tested under the following form:

\[ E_{t} \Delta c_{t+1} = \delta + \sum \gamma^{x} (i_{t} - E_{t} \pi_{t+1}) + \varepsilon_{t} \]

with a constant (\( \delta \)), coefficients (\( \gamma^{x} \)) and errors (\( \varepsilon \)) to be estimated.

For sake of comparability, I estimated the model only with the monetary policy rate and I accept the hypothesis that this rate alone does not bring useful information in explaining expected consumption growth. This result summarizes those of appendix C as raising a challenge for monetary policy models using the Euler equation: this equation, as it is used in monetary policy models, is at odds with the data. The estimates for equation (6) with household specific rates are reported in Table 2.

\(^3\)Inflation is defined in accordance with consumption as the growth rate (\( \Delta \log \)) of the considered consumption bundle deflator.
Table 2: Euler equation models (6) (CES utility function with expectations measured by a VARX with economic regressors or survey data)

For the US (Table 2a), coefficients are significantly different from 0, except for the fedfunds and the rate on mortgages. The signs of the estimated coefficients are nevertheless economically puzzling: for instance, the interest rate on car loans and personal loans have opposite effects on consumption. Overall this estimation yields a negative rate combination in nominal terms which is in odds with the theory.

For France (Table 2b), the coefficient on the regulated savings rate is not statistically significant. Similarly to the US, the signs of the coefficients vary, which is not economically interpretable and the optimal rate combination is largely negative.

Are these Euler equations on consumption? On this model I perform three Fisher’s tests examining whether the estimated equation can be called Euler equation on consumption in which household specific rates play a role:

$T_1$ on the joint nullity of all coefficients as a simple check

$T_2$ on the cumulated nullity of interest rate coefficients to verify that this model effectively relates expected consumption with interest rates so that it can be called a Euler equation on consumption

$T_3$ on the joint nullity of interest rate coefficients (other than monetary policy) to verify that rates other than the monetary policy one bring useful information to the model

$T_4$ on the equality between coefficients estimated using the expectation VAR and assuming perfect expectations

These tests are reported in Table 2. For the US countries all null hypothesis but the last are rejected. For France, only at the 10% threshold can I reject the hypothesis that $\sum x \gamma_x = 0$.

---

4The constant is not significantly different from 0 either: in equation (6) $ln(\beta)$ and the variance term are of opposite sign and can add up to zero.
The first three tests imply that the estimated equation is a *Euler equation on consumption* in which household specific rates play a role.

As for the last test, it is conducted to verify how the VARX estimated *ex ante* influences the estimation in this second step. The test concludes that the two estimations are not statistically different.

**Interest rates comparisons** From equation (6), the previous Fisher’s tests allow me to compute

\[ \frac{E_t \Delta c_{t+1} - \delta}{\sum_x \gamma^x} + E_t \pi_{t+1} \quad \text{and} \quad \frac{\sum_x \gamma^x i_t^x}{\sum_x \gamma^x} \]  

(7)

The left hand side of the previous equation is the marginal rate of substitution between present and future consumption (hereafter *MRS* denoted \( \tilde{i}_t \)).

\[ \tilde{i}_t = \frac{E_t \Delta c_{t+1} - \delta}{\sum_x \gamma^x} + E_t \pi_{t+1} \]  

(8)

If the model held exactly, this rate would be equal to the optimal combination of actual rates returned by the estimation \( \tilde{i}_t \)

\[ \tilde{i}_t = \frac{\sum_x \gamma^x i_t^x}{\sum_x \gamma^x} \]  

(9)

Table 3 displays some comparative statistics for actual and Euler interest rates in real terms.

In the US, the *MRS* is not correlated with the Fedfunds (< 10%). The correlation with the optimal rate combination is much larger (62%). For France, correlation with the Pibor-Euribor can even be negative. With the optimal spread combination it reaches 34% when deflation is based on expected future inflation (correlations 2).

However for both countries, the statistical properties of both the optimal rate combination and the *MRS* are not consistent with the initial interest rates. In particular, they are negative and the *MRS* is much more volatile than the corresponding optimal rate combinations (Table 3). Comparisons of theses rates depicted in Figure 2 convey the same message.

I find similar results when expectations are built on survey data. Also allowing for autocorrelation of the residual and habit formation in the utility function does not markedly modify the outcome of the estimations.
Figure 2: Actual interest rates and $MRS$ with a CES utility function (in nominal terms)
\[ \begin{array}{c|c|c} \text{Portfolio} & i^a & i \\ \hline \hline \text{(a) US} & & \\ \hline \hline \text{(b) France} & & \\ \end{array} \]

1 test two transformations of actual nominal rates in real terms: correlation 1 corresponds to the rate defined by contemporaneous inflation \( i_1 - \pi_1 \) and correlation 2 uses expected future inflation \( i_1 - E_1 \pi_{1+1} \).

**MRS**

**Positivity constraint**

\[ \forall x \quad \gamma^x \]

\[ \gamma^x \]

**MRS**

**MRS**

\[ \begin{array}{c|c|c} \text{Portfolio} & i^a & i \\ \hline \hline \text{(a) US} & & \\ \hline \hline \text{(b) France} & & \\ \end{array} \]

Reported values for the interest rate coefficients are the \( \gamma^x \), not the \( \lambda^x \). The variance matrix for the estimates is computed with the delta method and from the gradient of the likelihood at each date.
<table>
<thead>
<tr>
<th></th>
<th>Pollock</th>
<th>Opt. Cost</th>
<th>MRS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$i^{\text{pol}}$</td>
<td>$i^{\text{opt}}$</td>
<td>$i$</td>
</tr>
<tr>
<td>M fits</td>
<td>0.46</td>
<td>1.45</td>
<td>1.46</td>
</tr>
<tr>
<td>Std deviation</td>
<td>0.72</td>
<td>0.69</td>
<td>1.67</td>
</tr>
<tr>
<td>Minimum</td>
<td>-0.65</td>
<td>-0.6</td>
<td>-1.25</td>
</tr>
<tr>
<td>Maximum</td>
<td>2.76</td>
<td>3.32</td>
<td>7.15</td>
</tr>
<tr>
<td>Corr M.sep 1</td>
<td>1</td>
<td>0.81</td>
<td>0.88</td>
</tr>
<tr>
<td>Corr M.sep 2</td>
<td>0.81</td>
<td>1</td>
<td>0.84</td>
</tr>
<tr>
<td>Corr M.sep 2</td>
<td>0.70</td>
<td>1</td>
<td>0.32</td>
</tr>
</tbody>
</table>

(a) US

<table>
<thead>
<tr>
<th></th>
<th>Pollock</th>
<th>Opt. Cost</th>
<th>MRS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$i^{\text{pol}}$</td>
<td>$i^{\text{opt}}$</td>
<td>$i$</td>
</tr>
<tr>
<td>M fits</td>
<td>0.74</td>
<td>1.22</td>
<td>1.23</td>
</tr>
<tr>
<td>Std deviation</td>
<td>0.73</td>
<td>0.75</td>
<td>1.23</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.5</td>
<td>-0.10</td>
<td>-1.13</td>
</tr>
<tr>
<td>Maximum</td>
<td>2.52</td>
<td>2.9</td>
<td>16.16</td>
</tr>
<tr>
<td>Corr M.sep 1</td>
<td>1</td>
<td>0.95</td>
<td>0.18</td>
</tr>
<tr>
<td>Corr M-Sep 2</td>
<td>0.96</td>
<td>1</td>
<td>0.19</td>
</tr>
<tr>
<td>Corr M-Sep 2</td>
<td>0.96</td>
<td>1</td>
<td>0.12</td>
</tr>
<tr>
<td>Corr M-Sep 2</td>
<td>0.96</td>
<td>1</td>
<td>0.11</td>
</tr>
</tbody>
</table>

(b) France

Table 5: Comparative statistics of the interest rates and the $MRS$ under positivity constraint (in real terms)

Conclusion

The monetary policy rate does not bring useful information to explain the intertemporal elasticity of substitution of consumption of the representative consumer both in the US and France. This paper points to a representative household reacting to the interest rate on car loans in the US. For France this rate is not identified.

This result may seem trivial as it is clear that households can not borrow directly from the Central Banker’s desk. What is less trivial however, is that the spread between households’ rate and the monetary policy instrument matters so much and shall not be relegated to the residual of the Euler equation. Models should account for financial market frictions faced by households and consider weaker transmission mechanism of the monetary policy to a major share of real final demand, households’ consumption.

Household specific rates explain part of the wedge between the $MRS$ and the monetary policy rate in the US; however there remain sizeable discrepancies which do not rehabilitate the Euler equation as a powerful modelling element. In other words, the challenge to monetary policy models raised by Canzoneri et al. (2007) remains unchanged.

This dead-end may be due to omitted variables (current income, demographic effects, unemployment...) in the Euler equation under its simplest form. Indeed, econometric investigations of the consumption-savings behaviour often put forward richer specifications (Aviat et al., 2007; Bardaji et al., 2014). In the present paper I find that the $MRS$ is extremely volatile. Similar estimations as those presented in this paper with habit formation suggest that allowing the utility of consumption to be smoother does not reduce the volatility of the $MRS$. In further developments of this project, I will investigate the extend to which the measurement of expectations used can influence this result.
Figure 3: Actual interest rates and $MRS$ with a CES utility function under positivity constraint (in nominal terms)
References


A Data and the VAR of expectations

A.1 Data

France I use Insee’s Quarterly National Accounts data for consumption per capita excluding durables, its deflator, GDP, disposable income. Data, including interest rates, are available since the early eighties but I restrict the sample to post 1986 for stationarity reasons as this sample includes only the end of the period of high and volatile inflation. Interest rate (Pibor-Euribor and household specific rates) are taken from Banque de France and the ECB. For household balances of opinion, I use the European harmonized Household Survey published for France by Insee. Population and unemployment data are also retrieved from Insee’s public database. I use

\footnote{Data are taken from the first publication of base 2010 (2014Q1) but I also estimated with similar results the model on the last publication of base 2005.}
quarterly data spanning from 1986Q1 to 2013Q4.

**US** I use data from the National Income and Product Account (NIPA) for consumption, GDP, disposable income and deflators. I restrict consumption to non-durable goods and services. Real variables are taken per-capita. Interest rates, population and unemployment data are taken from the Fred database. I also use the surveys of consumers by the university of Michigan. I use quarterly data spanning from 1972Q1 to 2013Q2.

### A.2 The VAR of expectations

For section 1 and appendix C, I estimate a VARX to model households’ expectations. This method is based on Fuhrer (2000) and Canzoneri et al. (2007). I restricted the specification of the expectation VAR to a VAR in level with 12 lags. However, I show in appendix C that the rejection of the Euler equation does not depend on the choice of a VAR model.

**US** I estimate a VAR model corresponding to the log of consumption in non-durables per capita and its deflator. I replicate the VARX of Fuhrer (2000) and Canzoneri et al. (2007) to model households’ expectations in the US: additional regressors are the fedfund rate, log GDP and log GDI per capita. However, I do not use the Journal of Commerce industrial materials commodity price index for all items and tested some balances of opinion from the Michigan survey without significant modifications.

I also estimate expectations from a VARX using only balances of opinion from the Michigan survey: the Index of Consumer Expectations, Index of Current Economic Conditions and balances of opinion related to financial situation (past and expected), oppportunity to purchase durables, vehicles or a house, and expected inflation in the next 12 months.

**France** I estimate a VAR model corresponding to the log of consumption in non-durables per capita and its deflator. This VAR model also contains 12 lags and is estimated from 1986Q1 to 2013Q4 in level. To this VAR I add the lags of the purchasing power of gross disposable income, of GDP per capita, the Pibor-Euribor and test some balances of opinion from household opinion survey which do not significantly impact the outcome of the VAR.

I also estimate expectations from a VARX using only balances of opinion from the European harmonized consumers survey: general economic situation, unemployment, financial situation, consumer prices, savings intentions, current saving capacity.
B The conditional log-normality hypothesis

The equation under scrutiny takes the following form:

\[(1 + \dot{u}_t)^{-1} = \beta E_t \left( \frac{U_c(t + 1)}{U_c(t)} \frac{1}{\Pi_{t+1}} \right) \]  

(10)

In the most simple case \( U_c(t) = C_t^{-\sigma} = e^{-\sigma c_t} \) with \( c_t = \log(C_t) \) \( \pi_t = \log(\Pi_t) \)

\[ E_t \left( \frac{U_c(t + 1)}{U_c(t)} \frac{1}{\Pi_{t+1}} \right) = E_t(\exp(-\sigma \Delta c_{t+1} - \pi_{t+1})) = E_t(\exp(a' y_{t+1})) \]  

(11)

with \( a = [-\sigma, -1]' \) and \( y_t = [\Delta c_t, \pi_t]' \).

I suppose that households’ expectations are formed through the previous VAR model with normal errors \( \nu_t \) and variance \( \Sigma \), so that \( y_{t+1} = E_t(y_{t+1}) + \nu_{t+1} \). I can then decompose \( E_t \exp(a' y_{t+1}) = \exp(a' E_t y_{t+1}) E_t \exp(a' \nu) \), in which the second term can be simplified

\[ E_t \exp(a' \nu) = \int \frac{1}{(2\pi)^{\frac{n}{2}} |\Sigma|^{\frac{1}{2}}} \exp \left( a' \nu - \frac{1}{2} \nu' \Sigma^{-1} \nu \right) d\nu \]

first change of variables

\[ \Sigma^{-1} = P' D P \] with \( P \in \mathcal{O}(n) \) and \( D \) diagonal

\[ P \nu = \mu \]
\[ \nu = P' \mu \]

\[ E_t \exp(a' \nu) = \int \frac{1}{(2\pi)^{\frac{n}{2}} |\Sigma|^{\frac{1}{2}}} \exp \left( a' P' \mu - \frac{1}{2} \mu' D \mu \right) d\mu \]

second change of variables

\[ x = \left( \frac{D}{2} \right)^{\frac{1}{2}} \mu - \left( \frac{D}{2} \right)^{\frac{1}{2}} Pa \]
\[ x' = \frac{1}{2} \mu' D \mu - a' P' \mu + \frac{1}{2} a' P' D^{-1} Pa \]

we then have

\[ E_t \exp(a' \nu) = \int \frac{1}{(2\pi)^{\frac{n}{2}} |\Sigma|^{\frac{1}{2}}} \exp \left( -x' \frac{a' \Sigma a}{2} \right) \frac{D}{2} \frac{1}{|x' - \frac{a' \Sigma a}{2}|} dx \]

\[ E_t \exp(a' \nu) = \exp \left( \frac{a' \Sigma a}{2} \right) = \exp \left( \frac{\sigma^2}{2} \text{var}(e^c) + \sigma \text{cov}(e^c, e^\pi) + \frac{1}{2} \text{var}(e^\pi) \right) \]

\[ E_t \left( \frac{U_c(t + 1)}{U_c(t)} \frac{1}{\Pi_{t+1}} \right) = E_t(\exp(a' y_{t+1})) = \exp(a' E_t(y_{t+1})) \exp \left( \frac{a' \Sigma a}{2} \right) \]
Although it avoids a first order approximation, this method still forms a strong assumption: second moments $\Sigma$ are constant. In particular, this assumption shuts down one channel for precautionary savings: when the environment is more uncertain, $\Sigma$ increases, which is equivalent to a decrease in the interest rate. This channel of uncertainty shocks could account for an important share of business cycle fluctuations (Bloom, 2009).

C Confirming Canzoneri et al. (2007) on French data

If the Euler equation holds and households are rational and forming their expectations through a VAR model, the following should hold under log-normality hypothesis (Appendix B):

$$i_t = \sigma E_t \Delta c_{t+1} + E_t \pi_{t+1} - \ln(\beta) - \left( \frac{\sigma^2}{2} \text{var}(\epsilon^c) + \sigma \text{cov}(\epsilon^c, \epsilon^\pi) + \frac{1}{2} \text{var}(\epsilon^\pi) \right)$$

with $E_t \Delta c_{t+1}$, $E_t \pi_{t+1}$ the expectations formed through a VAR model for consumption growth and inflation respectively and $\epsilon^c$, $\epsilon^\pi$ the errors of this VAR model.

Considering external habit formation or catching-up with the Joneses under a multiplicative form Abel (1990); Gali (1994) modifies the preceding system by adding the lagged value of $\Delta c_t$ in the Euler equation whose coefficient will depend on both $\sigma$ and the habit formation parameter $\gamma$. The theoretical link (13) between expected consumption and interest rate becomes:

$$i_t = \sigma E_t \Delta c_{t+1} - \gamma(1-\sigma)\Delta c_t + E_t \pi_{t+1} - \ln(\beta) - \left( \frac{\sigma^2}{2} \text{var}(\epsilon^c) + \sigma \text{cov}(\epsilon^c, \epsilon^\pi) + \frac{1}{2} \text{var}(\epsilon^\pi) \right)$$

with households’ utility a CES function of $\frac{c_t}{c_{t-1}}$.

It is possible to assess graphically the fit to the Euler equation as in Canzoneri et al. (2007) by assuming a value for $\sigma$, $\gamma$ and plotting actual interest rates with the marginal rate of substitution between present and future consumption, either in real or nominal terms.

Figures 4 depict such comparisons using consumption in non-durables, with and without habit formation. These figures exhibit similar features to the US case reproduced in Figure 5. First, these graphs show that adding habit formation in the utility function does not move the MRS closer to the actual monetary policy rate. Tables 6 display some statistics for both countries. On average, there are positive spreads between the MRS implied by Euler equations and the monetary policy rate. As Canzoneri et al. point out, these spreads "have been well documented in the literature on the equity premium puzzle and the risk free rate puzzle" and are not a strong argument against the Euler equation. However the correlations reported in these tables are weak, even though their sign is not clear cut and depend on the way actual nominal rates are converted in real terms. These correlations are outlined by Canzoneri et al. as a challenge for macroeconomists and in particular for monetary policy models.
Figure 4: Pibor and MRS with a CES utility function ($\sigma = 2$) with and without habit formation ($\gamma = 0.6$)

<table>
<thead>
<tr>
<th></th>
<th>Pibor</th>
<th>CES without habit</th>
<th>CES with habit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.71</td>
<td>1.24</td>
<td>1.43</td>
</tr>
<tr>
<td>Std deviation</td>
<td>0.73</td>
<td>0.41</td>
<td>0.54</td>
</tr>
<tr>
<td>Minimum</td>
<td>-0.46</td>
<td>0.3</td>
<td>0.11</td>
</tr>
<tr>
<td>Maximum</td>
<td>2.33</td>
<td>3.15</td>
<td>3.55</td>
</tr>
<tr>
<td>Correlation1</td>
<td>1</td>
<td>0.15</td>
<td>0.16</td>
</tr>
<tr>
<td>Correlation2</td>
<td>1</td>
<td>-0.01</td>
<td>-0.01</td>
</tr>
</tbody>
</table>

(a) US

(b) France

1. I test two transformations of actual nominal rates in real terms: correlation 1 corresponds to the rate deflated with contemporaneous inflation $i_t - \pi_t$ and correlation 2 uses expected future inflation $i_t - E_t \pi_{t+1}$.

Table 6: Statistics for the actual and MRS (in real terms)

Figure 5 confirms Canzoneri et al.’s qualitative results for the US: the MRS markedly departs from the monetary policy rate "during the Voleker tightening in the early 1980s" and "during the Greenspan easing in the early 2000s". In addition to the mid 1990s, the MRS levelled off while actual rate remained stable. Also, in 2008 consumption nose dived more drastically than the interest rate would have implied: in nominal terms both rates plummeted, but in real terms MRS dropped while the real monetary policy rate peaked.

Figure 4 shows similar mismatch for France: the MRS mirrored the monetary policy rate in the 1990s and 2000s. In 2008, the French rates show the same disconnection in real and nominal terms as in the US.

Robustness to the calibration and the definition of the consumption bundle

Canzoneri et al. (2007) show that their results are robust to the definition of the utility function (with and without habit formation, either internal or external, additive or multiplicative) they also consider recursive utility and liquidity constraints (Campbell and Mankiw, 1990). As another robustness test, I compute similar MRS using other definitions of the consumption bundle and other calibrations of the utility function. Additional results are depicted on Figures 6 with
Figure 5: US Fed fund and MRS with a CES utility function ($\sigma = 2$) with and without habit formation ($\gamma = 0.6$)

A CES utility function without habit formation. These figures show, for three standard values of the intertemporal elasticity of substitution (2, 1, 0.5), the MRS computed with the VARX of expectations using four definitions of the consumption bundle (total consumption, consumption excluding durables, excluding energy or excluding committed expenditures (Quinet and Ferrari, 2008)). These interest rates are compared with the 3 month interbank rate and four different household specific rates, deposits and loans with various maturities (see section 1). Comparisons are made both in real and nominal terms. Tables 7 display the correlation matrices of the actual and MRS in real terms.

Figure 6a, 6c and 6e show that in nominal terms, the MRS from the Euler equations does not greatly depend on the consumption bundle considered, at least not in a way which would solve Canzoneri et al.'s challenge. Also they are more volatile than actual interest rates, all the more so when $\sigma$ is large.

In real terms, deflated by actual inflation, the larger volatility of the MRS is not clear cut (figure 6b, 6d and especially 6f). However, there remains a sizeable discrepancy between the levels of the MRS and interest rates set by the market in the eighties up to the mid-nineties. This stylized fact has often been interpreted in France in line with the liberalisation of financial markets over these years.

For each calibration, the interest rates can be split in two groups based on the correlations (Tables 7): MRS and actual interest rates. Even though the correlations between these groups can be sizeable (up to 50% for the liqret), the correlations within these groups are much larger and often close to one. These results are in line with Canzoneri et al.'s conclusions.

In addition to testing several definitions of consumption, I tested 3 VAR models to measure households expectations: one in first difference with two lags, one with additional regressors
\[(\sigma = 0.5)\]

| Euler eq. rate total consumption | 1  | -  | -  | -  | -  | -  | -  | -  | -  | -  |
| cons. excl. energy              | 0.86 | 1  | -  | -  | -  | -  | -  | -  | -  | -  |
| cons. excl. durables            | 0.94 | 0.78 | 1  | -  | -  | -  | -  | -  | -  | -  |
| cons. excl. committed exp.      | 0.94 | 0.83 | 0.86 | 1  | -  | -  | -  | -  | -  | -  |
| Pibor                           | 0.22 | -0.03 | 0.28 | 0.19 | 1  | -  | -  | -  | -  | -  |
| Deposit1y                       | 0.22 | -0.05 | 0.29 | 0.18 | 0.99 | 1  | -  | -  | -  | -  |
| Livret                          | 0.5  | 0.07 | 0.53 | 0.47 | 0.62 | 0.64 | 1  | -  | -  | -  |
| Mortgage                        | 0.27 | 0.01 | 0.36 | 0.21 | 0.94 | 0.96 | 0.66 | 1  | -  | -  |

\[(\sigma = 1)\]

| Euler eq. rate total consumption | 1  | -  | -  | -  | -  | -  | -  | -  | -  | -  |
| cons. excl. energy              | 0.87 | 1  | -  | -  | -  | -  | -  | -  | -  | -  |
| cons. excl. durables            | 0.93 | 0.78 | 1  | -  | -  | -  | -  | -  | -  | -  |
| cons. excl. committed exp.      | 0.94 | 0.87 | 0.84 | 1  | -  | -  | -  | -  | -  | -  |
| Pibor                           | 0.17 | -0.06 | 0.27 | 0.13 | 1  | -  | -  | -  | -  | -  |
| Deposit1y                       | 0.16 | -0.08 | 0.26 | 0.12 | 0.99 | 1  | -  | -  | -  | -  |
| Livret                          | 0.31 | -0.06 | 0.37 | 0.31 | 0.62 | 0.64 | 1  | -  | -  | -  |
| Mortgage                        | 0.21 | -0.02 | 0.34 | 0.15 | 0.94 | 0.96 | 0.66 | 1  | -  | -  | -  |

\[(\sigma = 2)\]

| Euler eq. rate total consumption | 1  | -  | -  | -  | -  | -  | -  | -  | -  | -  |
| cons. excl. energy              | 0.89 | 1  | -  | -  | -  | -  | -  | -  | -  | -  |
| cons. excl. durables            | 0.92 | 0.78 | 1  | -  | -  | -  | -  | -  | -  | -  |
| cons. excl. committed exp.      | 0.92 | 0.9  | 0.8 | 1  | -  | -  | -  | -  | -  | -  |
| Pibor                           | 0.13 | -0.07 | 0.24 | 0.08 | 1  | -  | -  | -  | -  | -  |
| Deposit1y                       | 0.11 | -0.1  | 0.23 | 0.06 | 0.99 | 1  | -  | -  | -  | -  |
| Livret                          | 0.16 | -0.14 | 0.22 | 0.18 | 0.62 | 0.64 | 1  | -  | -  | -  |
| Mortgage                        | 0.16 | -0.04 | 0.3  | 0.1  | 0.94 | 0.96 | 0.66 | 1  | -  | -  | -  |

Table 7: Correlation matrix of Euler equation and actual interest rates in real terms

identical to those in the benchmark models of this paper, one in over the year growth rate with the same regressors and households survey data.\(^6\) The present results are robust to these changes in the VAR model.

---

\(^6\) Balances of opinion on households survey data match the evolution of macroeconomic data over the year, in particular gross disposable income, better than over the quarter.
Figure 6: (FR) Actual interest rates and MRS for 4 definitions of the consumption bundle with a CES utility function without habit formation
Analyse économique des politiques française et allemande

Macro-economic import functions with imperfect competition - An application to the E.C. Trade

Les échanges internationaux de services de la France dans le cadre des négociations multilatérales du GATT

Le commerce extérieur et l'environnement international dans le modèle AMADEUS (révision 1992)

Coordination within the firm and endogenous growth

Innovation, taille et concentration : causalités et dynamiques

Investissement, incertitude et irréversibilité : quelques développements récents de la théorie de l'investissement

Les échanges internationaux de services de la France dans le cadre des négociations multilatérales du GATT

Quelle est l'influence du coût du travail sur l'emploi ?


Le modèle AMADEUS - Troisième partie : Le commerce extérieur et l'environnement international

Le choix de breveter une invention

Le modèle AMADEUS - Première partie : Présentation générale

Le modèle AMADEUS - Deuxième partie : Propriétés variantielles

Endogenous growth and product innovation

Aménésie des politiques françaises et allemandes : une analyse des politiques économiques

Irreversible Investment and Uncertainty: When is there a Value of Waiting?

Irreversible Investment and Uncertainty: When is there a Value of Waiting?

Technologie, changement organisationnel, qualifications et emploi : une étude empirique sur l'industrie manufacturière

Les liens entre RMI et chômage, une mise en perspective

Technologie, changement organisationnel, qualifications et emploi : une étude empirique sur l'industrie manufacturière

Investir dans la recherche et le développement : un examen sur match de worker-firm panel data

Deux études sur l'âge de la retraite

Investissement en recherche-développement : analyse de causalités dans un modèle d'accélérateur généralisé

Profitabilité de l'investissement productif et transmission des chocs financiers

Les apports de l'économie industrielle pour définir la stratégie économique de l'hôpital public

L'effet de la parenthèse de l'investissement sur les coûts de fonctionnement des entreprises

Le modèle AMADEUS - Deuxième partie - Propriétés variantielles

Innovation organisationnelle, dynamisme technologique et performances des entreprises

La théorie coopérative de la firme

Les apports de l'économie industrielle pour définir la stratégie économique de l'hôpital public

Les implantations japonaises en France

Technologie, changement organisationnel, qualifications et emploi : une étude empirique sur l'industrie manufacturière

La théorie coopérative de la firme

Négociations multilatérales du GATT

Les théories sur la structure optimale du capital : quelques points de repère

Les échanges internationaux de services de la France dans le cadre des négociations multilatérales du GATT

Techniques d'investissement et de financement du capital

Technologie, changement organisationnel, qualifications et emploi : une étude empirique sur l'industrie manufacturière

La théorie coopérative de la firme

Technologie, changement organisationnel, qualifications et emploi : une étude empirique sur l'industrie manufacturière

Technologie, changement organisationnel, qualifications et emploi : une étude empirique sur l'industrie manufacturière

Technologie, changement organisationnel, qualifications et emploi : une étude empirique sur l'industrie manufacturière

Technologie, changement organisationnel, qualifications et emploi : une étude empirique sur l'industrie manufacturière

Technologie, changement organisationnel, qualifications et emploi : une étude empirique sur l'industrie manufacturière
How does fuel taxation impact new car purchases? An evaluation using French consumer-level dataset

Durée passée en carrière et durée de vie en retraite : quel partage des gains d’espérance de vie ?

The walking dead Euler equation
Addressing a challenge to monetary policy models