MELEZE: A DSGE model for France within the Euro Area

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Mélèze : une modélisation DSGE de la France au sein de la zone euro

Résumé

Mélèze (Modèle économique linéarisé d’équilibre en zone euro) est un modèle néo-Keynésien de type DSGE avec les caractéristiques suivantes: la France et le reste de la zone euro forment une union monétaire; une fraction des ménages est non-ricardienne et consomme son revenu courant; les entreprises sont en concurrence monopolistique sur le marché des biens, et les travailleurs le sont sur le marché du travail; les prix et salaires sont rigides; les biens de consommation et d’investissement sont exportés/importés librement, tandis que le travail et le capital sont immobiles.

Cet article détaille la résolution et la calibration du modèle ainsi que sa linéarisation. En particulier, nous définissons l’état stationnaire du modèle en niveau et nous explicitons l’ensemble des contraintes de long-terme pesant sur la calibration du modèle. Dans un deuxième temps, nous présentons le comportement du modèle en réponse à divers chocs transitoires.

Mots-clés : modèle DSGE, union monétaire

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Abstract

MELEZE, standing for Modèle économique linéarisé d’équilibre en zone euro (linearised economic model of equilibrium in the euro area), is a new Keynesian DSGE model with the following characteristics: France and the rest of the Euro area form a monetary union; they are populated by infinitely lived households, of which a constant fraction is non Ricardian, consuming all of their current income; firms operate in monopolistic competition on the goods market, and so do workers on the labour market indistinctly of their financial constraints; prices and wages are sticky; consumption and investment goods can be freely exported/imported, whereas workers and installed capital cannot.

The present paper presents the resolution and the calibration of the model as well as its full linearisation. In particular, we characterize the unique steady state in levels for the real variables and explicit the induced constraints on the parametrisation. In a second part, we present the behaviour of our model with respect to standard transitory shocks.

Keywords: DSGE model, monetary union

Classification JEL : E10, F45
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1 Introduction

Economic policy analysis at the Insee is traditionally focused on the use of large-scale macroeconometric models namely Mésange (Klein and Simon, 2010). The Mésange model constructed on a highly detailed macroeconomic accounting framework in line with Quarterly National Accounts, features around 500 equations, 10% being behavioural econometric relationships. Its core long term economic structure is neo-classical, whereas the model includes real and nominal rigidities in the short run. One main advantage of this model is its ability to replicate past observed data and to give precise and quantitative insights for economic policy evaluations, mainly focused on fiscal evaluations.

However, Lucas’ critique (Lucas, 1976) states that the absence of a strong short-run economic structure and the absence of rational expectations might be weaknesses of such models. As such, economic modelling has focused on alternatives and developed a new modelling stream, namely the DSGE literature. Contrary to macroeconometric models, these models focus less on giving an exact description of observed economic data than on a strong theoretical and micro-founded structure answering Lucas’ critique. As such, they appear as an interesting parallel to macroeconometric models, as evidenced by their large adoption today within the main national and international institutions.

This paper introduces MELEZE\(^1\), a fiscal-policy oriented new-Keynesian Dynamic Stochastic General Equilibrium (DSGE) model developed at the Insee. MELEZE is a two-country monetary union model aimed at representing the situation of France within the Euro area.

Compared to the Mésange model, the present model allows to focus more specifically on questions related to rational expectations and anticipations of shocks, macroeconomic spillovers in a monetary union, and endogenous behaviours of the government. In particular, MELEZE offers two alternative choice of modelling for the government behaviour. A first option is the implementation of a traditional budget rules linking today’s public consumption to past deficits (as found in the institutional DSGE literature). As an alternative, we also develop and propose a new approach, modelling the government as an optimizing agent maximizing households’ welfare. This second modelling directly relates to the Ramsey policy under bounded rationality.

More generally, MELEZE’s structure compares with most standard tools developed in international institutions and central banks, yet remains simple enough to be able to disentangle most economic reactions. Indeed, the construction of the model was conducted along three objectives. First, it needs to be refined enough to give insightful supplementary policy analyses using a DSGE model rather than a macroeconometric model, in particular with respect to fiscal behaviours. Second, it should compare to

\(^1\) Modèle Économique Linéarisé d’Équilibre en Zone Euro
standard institutional models used at central banks and international organizations and help to bridge the gap between Mésange and such large-scale DSGE models. Last, it should remain relatively small compared to large institutional DSGE models to both allow for a better understanding of underlying mechanisms and for an easier transmission of the model internally.

Along those lines, MELEZE therefore includes all traditional ingredients found in the institutional DSGE literature, and is largely inspired by (Smets and Wouters, 2002, 2003, 2005) or (Christiano, Eichenbaum, and Evans, 2005).

All in all, these modelling choices allow to perform in MELEZE, all standard macroeconomic policy evaluation exercises such as the study of permanent fiscal reforms (Campagne and Poissonnier, 2016a), or the simulation of labour and goods markets deregulations (Campagne and Poissonnier, 2016b).

This paper is organised as follows. In Section 2, we present the model and derive the first order conditions for all economic agents. Section 3 describes the methodology and the data used for the calibration of MELEZE as a monetary union model describing the situation of France within the Eurozone. Lastly, in order to give a few insights on the behaviour of our model, we present the short term responses through the study of impulse responses to a set of standard policy shocks (Section 4).

The Appendix gives a full description of the steady state and linearised model, as well as additional remarks on the design of monetary policy in a currency union, on the impact of wage and price dispersion in the model and on the uniqueness and stability of the steady state.

2 Model

Non-technical outline  In this paper, we develop a DSGE model comparing with standard tools developed in international institutions and central banks (Christiano et al., 2005; Smets and Wouters, 2003). The model consists of two countries where continuums of firms and households interact on the goods, labour and capital market. Both firms and households are considered immobile across countries.

As advocated by Mankiw (2000), we distinguish between two types of households. A fraction of these households is Ricardian, that is not financially constrained. They hold financial asset (or debt), own capital which they lend to firms in their country (once installed capital is assumed to become immobile) and also own financial intermediation firms. Therefore, they receive (or pay) interests and dividends. These Ricardian households also choose their investment each period by arbitrating between capital and the risk free asset. Non Ricardian households on the contrary are financially constrained and
do not hold any asset.

Both types of households also provide labour on monopolistically competitive market. For this reason, households are paid with a mark-up over their marginal disutility. Wage rigidities are added over the cycle, and each household can only reset its wage in adequateness with his optimal consumption-leisure arbitrage with an exogenous probability. In this framework, there is no involuntary unemployment and labour adjusts only at the intensive margin (hours worked).

Households finally consume both domestic and imported goods which are partial substitutes. For Ricardian households, being non financially constrained allows them to smooth their consumption over time. Non Ricardian households on the contrary can not. Once their wage level is set, their labour supply is given by firms demand, their income ensues which they consume entirely within the same quarter.

Firms produce partially substitutable goods from a standard constant returns to scale production function. Production factors are labour and capital. Total factor productivity is exogenous and growing at the same pace across countries. At each period firms optimize their relative demand in capital and labour to minimize their production cost, taking the aggregate wage and gross return on capital as given. Partial substitutability allows firm to price a mark-up over their marginal cost. Over the cycle, with an exogenous probability each firm can reset its price to maximize its expected discounted profits, while internalising its market power. Those price rigidities lead to a New Keynesian Phillips curve.

The modelling of governments’ behaviour departs from the fiscal and budget rules literature used in quantitative models to endogenize tax rates and public spending in order to ensure the government’s budget constraint. We consider here forward-looking optimizing governments. We introduce unproductive public consumption as a proxy for actual public spendings, public investment, public employment and production of public services altogether. This consumption enters households utility function together with private consumption. Governments maximize the intertemporal households utility under the public budget constraint which is an approximation for the exact Ramsey problem. Alternatively, we also allow the government to behave according to a standard budget rule linking public consumption with past deficit and output gap. Furthermore, governments collect taxes on wages, capital income, dividends, consumption and investment. They can distribute transfers to both types of households. They also hold debt both at the steady state and over the cycle.

In addition to production of real goods by the firms, a union wide financial market produces financial intermediation services both for households and governments. Financial intermediaries capture on top of the interest rate set by the central banker a fee under the form of a debt elastic spread which is akin to fisim. There are no risk or agency issues in our model so that this fee is not to be interpreted as
a risk premium of any kind. In practice, these financial intermediaries ensure the closing of the model as exposed in Schmitt-Grohé and Uribe (2003) and have a very small production compared to firms of the real sector.

Notations As much as possible, we keep standard notations throughout this paper (C for consumption, W for wage...). A superscript \( i \in \{1, 2\} \) whether on an aggregate or on a parameter refers to the country. Subscripts are used to specify an operation related to the variable (e.g. habit on consumption or labour), in particular \( C_j^i \) refers to consumption in country \( i \) of goods produced in country \( j \). Upper-case letters refer to aggregates while lower-case letters refer per GDP unit aggregates or sometimes when we want to emphasize individual variables (wage, labour and output per firm or capita). Throughout, \( \tau \) is the index for a generic household and \( \varepsilon \) the index for a generic firm. R and NR superscripts relate to Ricardian and non Ricardian households respectively. \( t \) refers to time.

A full dictionary of variables and parameters is given in Tables 3 and 4 in the Appendix.

2.1 Goods and labour aggregations

2.1.1 Aggregation of production within countries

We assume that a continuum of goods of size \( P \) is produced in the monetary union. Goods in \([0, pP]\) are produced in country 1, while goods in \([pP, P]\) are produced in country 2. For formula generalization, we shall denote \( p^1 = p \) and \( p^2 = 1 - p \).

In each country, domestic production is aggregated into a domestic good using a Dixit–Stiglitz aggregator (Dixit and Stiglitz, 1977) with an elasticity of substitution specific to each country. This modelling hypothesis is interpretable either as the technology of a perfectly competitive final good sector with sole inputs intermediate consumption of the continuum of firms or as the relative preferences for each type of goods of the final consumer. These hypotheses yield the following relationship between the demand for goods produced by firm \( y^i(\varepsilon, i) \) and the total demand for production of country \( i \) \((Y^i_t)_{i \in \{1, 2\}}\):

\[
Y^1_t = Z_1 \left( \frac{1}{pP} \int_0^{pP} y^1(\varepsilon) \frac{\rho^1}{\sigma^1} d\varepsilon \right)^{\frac{1}{\sigma^1-1}} = \left( \int_0^{pP} y^1(\varepsilon) \frac{\rho^1}{\sigma^1} d\varepsilon \right)^{\frac{1}{\sigma^1-1}}, \tag{2.1}
\]

\[
Y^2_t = Z_2 \left( \frac{1}{(1-p)P} \int_{pP}^{P} y^2(\varepsilon) \frac{\rho^2}{\sigma^2} d\varepsilon \right)^{\frac{1}{\sigma^2-1}} = \left( \int_{pP}^{P} y^2(\varepsilon) \frac{\rho^2}{\sigma^2} d\varepsilon \right)^{\frac{1}{\sigma^2-1}}. \tag{2.2}
\]
where $\theta^i$ is the elasticity of substitution between goods in country $i$ and $Z_i$ a constant of normalisation.  

Maximising consumption under the budget constraint or alternatively, minimising the price for a bundle unit yields the corresponding production prices:

$$P^1_t = \frac{1}{Z_1} \left( \frac{1}{(p/p')^{\theta^1}} \int_0^{p/P} P^1_t(\epsilon)^{1-\theta^1} d\epsilon \right)^{1/\theta^1} = (p/P)^{1/\theta^1} \left( \int_0^{p/P} P^1_t(\epsilon)^{1-\theta^1} d\epsilon \right)^{1/\theta^1}, \quad (2.3)$$

$$P^2_t = \frac{1}{Z_2} \left( \frac{1}{((1-p)/p')^{\theta^2}} \int_{p/P}^{P} P^2_t(\epsilon)^{1-\theta^2} d\epsilon \right)^{1/\theta^2} = ((1-p)/P)^{1/\theta^2} \left( \int_{p/P}^{P} P^2_t(\epsilon)^{1-\theta^2} d\epsilon \right)^{1/\theta^2}. \quad (2.4)$$

The resulting relationships between aggregated and retail prices and quantities read:

$$y^1_t(\epsilon) = \frac{Z^1}{Z_1} \left( \frac{1}{p/P} \right)^{\theta^1} \left( \frac{P^1_t(\epsilon)}{P^1_t} \right)^{-\theta^1} \frac{Y^1_t}{P^1_t}, \quad (2.5)$$

$$y^2_t(\epsilon) = \frac{Z^2}{Z_2} \left( \frac{1}{(1-p)/P} \right)^{\theta^2} \left( \frac{P^2_t(\epsilon)}{P^2_t} \right)^{-\theta^2} \frac{Y^2_t}{(1-p)/P}. \quad (2.6)$$

### 2.1.2 Labour aggregation

The aggregation of labour in both countries is symmetric to that of goods (Dixit–Stiglitz). The population size is set to $N$ and a share $\tau$ of households, that is households in $[0,\tau N]$, live in country 1. Households in $[\tau N, N]$ live in country 2. For formula generalization, we shall denote $n^1 = \tau$ and $n^2 = 1 - \tau$. Labour is hereafter assumed to be immobile across countries.

Household $\tau$ supplies labour $l^i(\tau, t)$ and demands the wage $w^i(\tau, t) so that the total supply of labour and average wage in country $i$ are $(L^i_t)_{i=(1,2)}$ and $(W^i_t)_{i=(1,2)}$.

$$L^1_t = (\tau N)^{1-\theta^1} \left( \int_0^{\tau N} l^1_t(\tau, t) d\tau \right)^{\theta^1}.$$

$$L^2_t = ((1-\tau)N)^{1-\theta^2} \left( \int_{\tau N}^{N} l^2_t(\tau, t) d\tau \right)^{\theta^2}.$$

where $\theta^i$ is the elasticity of substitution between labour types in country $i$.

---

2We take $Z_1 = p/P$ and $Z_2 = (1-p)/P$ to simplify the algebra. With this normalisation, if prices $P^i_t(\epsilon)$ are all equal within country 1, then the aggregate price index is equal to the individual price $P^1_t = P^1_t(\epsilon)$. In addition, each firm produces an equal share of total output $y^1_1(\epsilon) = \frac{1}{P_t} Y^1_t$. 

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The corresponding wages are:

\[
W^1_t = \left( nN \right)^{\frac{1}{\theta_1(1-\theta_1)}} \left( \int_0^{nN} w^1_1(\tau)^{1-\theta_1} d\tau \right)^{\frac{1}{1-\theta_1}},
\]

\[
W^2_t = \left( (1-n)N \right)^{\frac{1}{\theta_2(1-\theta_2)}} \left( \int_n^{N} w^2_2(\tau)^{1-\theta_2} d\tau \right)^{\frac{1}{1-\theta_2}}.
\]

The resulting relationships between aggregated and individual wage and labour supply read:

\[
l^1_1(\tau) = \frac{1}{nN} \left( \frac{w^1_1(\tau)}{W^1_t} \right)^{\frac{1}{\theta_1}} L^1_t,
\]

\[
l^2_2(\tau) = \frac{1}{(1-n)N} \left( \frac{w^2_2(\tau)}{W^2_t} \right)^{\frac{1}{\theta_2}} L^2_t.
\]

2.1.3 Aggregation of domestic and imported private consumption

In both countries, households have access to goods produced by each country; domestic and foreign goods are partial substitutes. We derive the case for consumption, we assume that investment and consumption goods are identical so that the same results apply to investment as well. Aggregation of imported and domestic production along with the associated consumption price index are modelled as follows

\[
C^i_t = \frac{C^i_{1,t}^{1-a^i} C^i_{2,t}^{a^i}}{(1-a^i)^{1-a^i} a^i}
\]

\[
CPI^i_t = P^i_t^{1-a^i} P_j^{a^i}
\]

where \( C^i_t \) is the private consumption of country \( i \) and \( C^i_{j,t} \), is the private consumption in country \( i \) of the aggregated goods produced in country \( j \). \( a^i \) is the import share of country \( i \) and therefore a measure of trade openness.

Note that we have the following relationships:

\[
C_{i,t} = C^i_{1,t} + C^i_{2,t}
\]

\[
C^i_t \neq C^i_{1,t} + C^i_{2,t} \quad \text{but} \quad CPI^i_t = \frac{P^i_t P_j}{P^i_t}
\]

The aggregation of domestic and imported consumption in both countries yields the following relationships:

\[
C^2_{2,t} = a^1 \left( \frac{P^1_t}{P^2_t} \right)^{1-a^1} C^1_t \quad C^1_{1,t} = (1-a^1) \left( \frac{P^1_t}{P^2_t} \right)^{a^1} C^1_t
\]

\[\text{(2.17, 2.18)}\]
\[ C_{t,1} = \alpha^2 \left( \frac{p_{t,2}}{p_{t,1}} \right)^{1-a^2} C_t^2 \quad C_{t,2} = (1 - \alpha^2) \left( \frac{p_{t,1}}{p_{t,2}} \right)^{a^2} C_t^2 \] (2.19, 2.20)

The repartition of consumptions between locally-produced goods and foreign ones depends on the degrees of openness (conveyed by import shares \( \alpha^i \)). Imported and domestic consumption in country 1 respond to the terms of trade defined as \( T_t = \frac{p_{t,2}}{p_{t,1}} \), with elasticities \( \alpha^1 - 1 \) and \( \alpha^1 \), respectively: as expected, the dearer are import prices in relative terms, the more households consume domestically-produced goods.

### 2.2 Households

#### 2.2.1 Consumption and investment decision of Ricardian households

In both countries, we assume that a fraction \( (1 - \mu^i) \) of households can participate to the financial markets. These households can borrow or lend money on an international market (see 2.5) and doing so have the possibility to smooth their consumption across periods. Each household of this type \( (\tau) \) maximises her intertemporal utility function subject to her budget constraint (determined by the recursive law of motion of private assets). Utility is similar to Trabandt and Uhlig (2011), that is non separable, CES in consumption with external habit formation in a multiplicative manner.\(^3\) Disutility of labour also allows for habits.\(^4\) This functional form is compatible with long term growth (King, Plosser, and Rebelo, 2002), as under this form the disutility of labour is concave for any value of the intertemporal elasticity of substitution of consumption, and also ensures a constant Frisch elasticity.

We also consider that households derive utility from public expenditure and therefore redefine the consumption bundle as a combination of private and public expenditures in a Cobb-Douglas fashion. This choice of specification is a subcase of CES aggregations as in McGrattan, Rogerson, and Wright (1997); Bouakez and Rebei (2007); Coenen, Mohr, and Straub (2008).

In all, a Ricardian household \( \tau \) solves:

\[
\max_{C^{i}(r),I^{i}(r),K^{i}(r),L^{i}(r)} E_t \sum_{t=1}^{\infty} \beta^{T-t} U(C^{R,i}(r), C^{i}_{T-1}) V(I^{R,i}(r), L^{i}_{T-1}) W(G^{i}(r), G^{i}_{T-1})
\] (2.21)

\(^3\)As in Abel (1990); Galí (1994); Carroll, Overland, and Weil (2000); Fuhrer (2000), introducing habits allows to account for the persistence of consumption when estimating the model. It also allows for more realistic hump-shaped response functions following shocks.

\(^4\)Note however, that in the standard calibration of the model, these habits on labour are muted.
subject to the budget constraint

$$FA^i_T(\tau) = \left( R^i_{T-1} - \psi \left( \frac{FA^i_{T-1}}{P^i_{T-1}} \right) \right) FA^i_{T-1}(\tau) + w^i_T(\tau) i^R_j(\tau)$$

$$- CPI^i_T (1 + \nu^c_j) C^R_j (\tau) + (1 - \nu^D_j) D^i_T(\tau) + (1 - \nu^FD_j) FD^i_T(\tau)$$

$$+ \Phi^i_T (\tau) + (1 - \nu^K_j) CPI^i_T K^j_{T-1}(\tau) - CPI^i_T (1 + \nu^c_j) I^i_T(\tau)$$

(2.22)

and the capital accumulation equation

$$K^i_T(\tau) = (1 - \delta) K^i_{T-1}(\tau) + \epsilon^{i,l}_T \left[ 1 - S \left( \frac{L^i_T(\tau)}{L^i_{T-1}(\tau)} \right) \right] I^i_T(\tau)$$

(2.23)

Under the most general form, we define utility as:

$$U(C^R_j(\tau), C^i_{-1}) = V(L^i_T(\tau), L^{i'}_{-1}) W(G^i, G^i_{-1}) =$$

$$\left[ \left( \frac{C^R_j(\tau)}{G^i_{-1}} \right)^{-h^i_j} \right]^{-1} - h^i_j \left[ C^i_{-1} \left( \frac{C^i_{-1}}{G^i_{-1}} \right)^{-h^i_j} \right]^{-\eta}$$

$$\frac{1 - \sigma^i_j}{1 - \sigma^i_j} \left[ 1 - \kappa^i (1 - \sigma^i_j) \left( \frac{L^i_{-1}}{L_{-1}} \right)^{-h^i_j} \right]^{1 + \sigma^i_j}$$

(2.24)

$E_t$, $\beta^i$ are respectively the expectation at time $t$ operator and the discount factor; $C^R_j(\tau)$ is the consumption of Ricardian agent $\tau$ in country $i$; $C^i_{-1}$ is the inverse intertemporal elasticity of substitution. $\kappa^i$ is the weight assigned to labour in the utility function; $\sigma^i_j$ is the inverse of the Frisch elasticity. $h^i_j$, $h^i_j$, $h^i_j$ are the external habit formation parameters on (private and public) consumptions and labour. $L^i_{-1}$ is the labour supply of household $\tau$ and $w^i_T(\tau)$ its wage.

$FA^i_T(\tau)$ is the household’s $\tau$ asset holdings at the end of period $t$ while $FA^i_j$ is country’s $i$ aggregate level of private financial assets (see the atomicity assumption explained in the following paragraph on private asset dynamics); $R_i$ is the interest rate set by the monetary authority in the union; $\psi$ is an interest premium on debt (whose function is detailed subsequently).

$\nu^c_j$ is the tax rate on consumption or value-added tax (VAT) through which government expenditure is partially financed. $D^i_T$ is the dividend paid by the firm to its owners taxed at rate $\nu^D_j$, $FD^i_T$ are equivalently the dividends paid by the financial sector taxed at rate $\nu^FD_j$ and $\Phi^i_T(\tau)$ a lump-sum transfer from the government. Finally, $K^i_T(\tau)$ is the capital stock of Ricardian households depreciating at rate $\delta$ and which revenues are taxed at rate $\nu^K_j$. 

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$\bar{Y}_i$ corresponds to the steady state level of output, and $Tr_{t-1} = (1 + g)^{t-1}$ corresponds to the deterministic trend of our model, where $g$ is the growth rate of TFP.

In the capital accumulation equation, $I^*_i(\tau)$ is the investment level with an adjustment cost\(^5\) $S \left( t_i(\tau)/t_{i-1}(\tau) \right)$ depending on previous period level of investment.\(^6\) As a result, households pay for the full investment allotment $I^*_i(\tau)$ and a share $S \left( t_i(\tau)/t_{i-1}(\tau) \right)$ is lost in the installation process. This adjustment cost mitigates the fluctuations of capital stock and investment in reaction to exogenous shocks. $\epsilon^i_t$ represents an exogenous shock to this cost sometimes found crucial to replicate the business cycle. The costate variable for constraint (2.23) is defined as $q_t = CPI_i t \left( 1 + \nu^c \right)$ times the costate variable of the budget constraint (2.22) $\beta^j t \lambda_t$, so that $q_t$ is the market value of an additional unit of capital, that is Tobin’s marginal $Q$.

The Euler equation, the investment decision and Tobin’s Q for this programme are identical across households, and under the assumption that differences in labour and consumption across Ricardian households are of second order, we aggregate these first order conditions.\(^7\)

The Euler equation describes the trade off between consumption and savings (on the financial market):

$$\begin{align*}
\beta^{j} E_t \left\{ \mathcal{U} \left( \frac{C^{i}_{t+1}}{(1 - \mu^t)^mN^t}, C^i_t \right) \mathcal{V} \left( \frac{L^{i}_{t+1}}{(1 - \mu^t)^mN^t}, L^i_t \right) \mathcal{W} \left( G^{i}_{t+1}, G^i_t \right) \right. \\
\left. \mathcal{U} \left( \frac{C^{i}_{t}}{(1 - \mu^t)^mN^t}, C^i_{t-1} \right) \mathcal{V} \left( \frac{L^{i}_{t}}{(1 - \mu^t)^mN^t}, L^i_{t-1} \right) \mathcal{W} \left( G^{i}_{t}, G^i_{t-1} \right) \right\} R_t - \Psi \left( \frac{FA^j_t}{1 + \psi_{t+1}} \right) = \frac{\Pi_{C^i, t+1}}{1 + \psi_{t+1}} \left( 1 + \psi_{t+1} \right) \right) = 1
\end{align*} \tag{2.25}$$

where $\Pi_{C^i, t+1}$ is the inflation of the consumption price index in country $i$.

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\(^5\)As in Smets and Wouters (2003) to Smets and Wouters (2007), this cost is introduce in order to smooth the reaction of investment to shocks. Similarly, we assume that at steady state $S = 0$, $S' = 0$ and $S'' > 0$.

\(^6\)As in most DSGE models, the investment decision is conducted by households only.

\(^7\)The extent of this approximation is explained by Carroll (2000) in a more general context. Within a linearised model we give a detailed analysis of distributions in Appendix D.
Investment and the marginal value of capital are described by the following first order conditions:

\[
1 = q_i^t e^{l,t} \left\{ 1 - S \left( \frac{I_i^t}{I_{t-1}^t} \right) - S' \left( \frac{I_i^t}{I_{t-1}^t} \right) \frac{I_i^t}{I_{t-1}^t} \right\} \\
+ \beta^t E_t \left\{ \frac{U'}{(1-p^t)\nu^i N} C_i^t \right\} \left\{ \frac{L_i^t}{(1-p^t)\nu^i N} C_i^t \right\} W(G_t^{i+1}, G_t^i) q_{i+1}^t e^{l,t} \left( \frac{I_{i+1}^t}{I_i^t} \right) \left( \frac{I_i^t}{I_{t-1}^t} \right)^2 \right\}
\]

(2.26)

\[
q_i^t = \beta^t E_t \left\{ \frac{U'}{(1-p^t)\nu^i N} C_i^t \right\} \left\{ \frac{L_i^t}{(1-p^t)\nu^i N} C_i^t \right\} W(G_t^{i+1}, G_t^i) \left( q_{i+1}^t (1 - \delta) + \frac{(1 - \nu_{i+1}^t \nu_{i+1}^t)}{1 + \nu_{i+1}^t} \right) \right\}
\]

(2.27)

The latter, similar to the Euler equation on consumption (2.25), describes the trade-off between investment in capital and consumption.

### 2.2.2 Ricardian households’ asset dynamics

By assumption, only unconstrained households can lend or borrow, their aggregate budget constraint reads:

\[
FA_i^t = \left( R_{i-1}^t - \psi \left( \frac{FA_{i-1}^t}{P_{i-1}^t \bar{Y}^t T_{t-1}^t} \right) \right) FA_{i-1}^t + W_{i}^{R_{i}^{K_{i}^{t}}} R_{i}^{K_{i}^{t}} \\
- CPI_t^i (1 + \nu_{i}^{c_{j}^{t}}) C_i^{R_{i}^{j}} + (1 - \nu_{i}^{D_{j}^{t}}) D_i^t + (1 - \nu_{i}^{FD_{j}^{t}}) FD_i^t \\
+ \Phi_i^{R_{i}^{j}} + (1 - \nu_{i}^{K_{i}^{t}}) CPI_t^i R_{i}^{K_{i}^{t}} K_{i-1}^t - CPI_t^i (1 + \nu_{i}^{c_{j}^{t}}) I_i^t
\]

(2.28)

To make the cost of debt increase with the level of indebtedness and also ensure the stationarity of the model (i.e. rule out unit roots to ensure the convergence of financial assets after shocks, see section E), we include a premium on the interest rate \( \psi \), which is akin to a transaction cost on holding assets paid to an international financial intermediary and enforces a no-Ponzi scheme condition on the evolution of assets (see Section 2.5). This premium depends positively on \( f_a_i^t = \frac{FA_i^t}{P_i^t Y_t^{1/\phi}} \), which represents the level of indebtedness of private agents in country \( i \) in real terms, \( Y_i \) being the steady-state value of output (once detrended) in country \( i \) and \( T_t \) its deterministic trend.

The premium a household faces depends on the aggregate private asset holdings of the country (or local financial conditions), not on the household’s private personal financial position. Thus each household takes the premium as given in its consumption decision (atomicity assumption). As the model
will be linearised, only the value of $\psi$ and its first derivative at the steady state will impact the model dynamics. We specify $\psi$ such that $\psi(0) = 0$ and $\frac{d\psi(x)}{dx} > 0$, so that both indebtedness and asset holding incur a cost paid to the intermediary, and the value of the premium increases with debt.

If at the aggregate level, households in country $i$ are net borrowers (i.e. $FA^i_t \leq 0$), resident households have to pay an interest premium on their debt amounting to $|\psi(fa^i_0)|$. When the country is net lender, returns are reduced by $\psi(fa^i_0)$ captured by the intermediary. This mechanism is equivalent to financial intermediation services indirectly measured (FISIM, see section 2.5).

2.2.3 Consumption decision of non Ricardian households

The remaining fraction $\mu^i$ of households does not have access to financial intermediation and therefore, their consumption cannot be smoothed across periods. These non Ricardian households follow a rule-of-thumb:

$$0 = w^i_t(\tau)l^i_t(\tau) + \Phi^i_t(\tau) - CPI^i_t(1 + \nu^{ij}_c)i^C_t$$

on aggregate $0 = W^{NR,i}_tN^{NR,j}_t + \Phi^{NR,j}_t - CPI^{NR,i}_t(1 + \nu^{ij}_c)i^{NR,j}_t$ (2.29)

As no assets are available to non Ricardian households, they neither hold shares in domestic firms nor in the financial sector and hence do not receive dividends. Moreover, in the absence of precautionary savings of these households in our model, non-Ricardian households cannot hold money as a partial substitute to bonds or capital and are unable to smooth their consumption in time even partially. As a consequence, our model may overestimate their reaction to shocks (Challe and Ragot). For instance following a positive productivity shock, prices and wages being sticky, firms will lower their labour demand. While Ricardian households can smooth their consumption by selling some assets, consumption of the non Ricardian ones will drop as a direct consequence of their decrease in payroll, this lower demand will in turn affect output negatively.

2.2.4 Labour supply decision and wage setting

As we did for consumption goods, we model labour aggregation with a Dixit–Stiglitz function. Relationships between labour and wages are therefore similar to those between consumption and prices (see section 2.1). Unlike consumption goods, labour is considered immobile and cannot be imported or exported. The relationship between total demand for labour and each household supply as a function of the demanded wage reads:

$$l^i_t(\tau) = \frac{1}{n_iN} \left( \frac{w^i_t(\tau)}{W^i_t} \right)^{-\theta^{\nu}_w} L^i_t$$

(2.31)
We assume wage stickiness à la Calvo (1983), with parameter $\tilde{\sigma}_{iw}$ denoting the probability not to adjust wages at each period. There is also partial indexation of wages on past inflation of consumption prices according to parameter $\gamma_{iw}$ and indexation on targeted inflation with parameter $1 - \gamma_{iw}$. Wages are also indexed on the deterministic trend of TFP.\(^8\) Households solve the following program:

$$\max_{\tilde{w}_{i,T}(\tau), \tilde{\beta}_{i,T}(\tau)} \sum_{t=0}^{\infty} \left( \tilde{C}_{i,T}^{\tau} \right)^{T-t} \mathcal{U}(C_{i,T}, C_{T-1}) \mathcal{V}\left( \tilde{h}_{i,T}(\tau), L_{T-1}^{i}\right) W(G_{i,T}, G_{T-1})$$

subject to the labour demand function:

$$\tilde{h}_{i,T}(\tau) = \frac{1}{\tilde{W}_{i}} \left( \frac{\tilde{W}_{i}^{j} \tilde{w}(\tau)}{\tilde{W}_{i}} \right)^{-\tilde{\sigma}_{iw}} L_{T},$$

as well as their respective budget constraint, and the following indexation rule:

$$\tilde{w}_{i,T}(\tau) = \tilde{w}_{i}(\tau) \prod_{k=t}^{T} \Gamma_{w,i}^{j}(\Pi_{T}^{i,j})^{1 - \gamma_{iw}} \frac{\tilde{T}_{T}}{T_{T}} = \tilde{w}_{i}(\tau) \Gamma_{T-1}^{T-1},$$

where $\tilde{w}_{i}(\tau)$ is the optimal wage set at time $t$ by household $\tau$ and $\tilde{w}_{i,T}(\tau)$ is its wage at time $T$ when not reset between time $t$ and $T$; $\tilde{h}_{i,T}(\tau)$ and $\tilde{h}_{i,T}(\tau)$ are the corresponding labour demands. $\Gamma_{w,i}^{T-1}$ denotes the indexation factor $\prod_{k=t}^{T-1} \Gamma_{w,i}^{j}(\Pi_{T}^{i,j})^{1 - \gamma_{iw}} \tilde{T}_{T}/T_{T}$ with $\Pi_{T}^{i,j}$ the steady state inflation of $CPI^{j}$.

The first order condition reads

$$0 = \tilde{E}_{i} \sum_{t=0}^{\infty} \left( \tilde{C}_{i,T}^{\tau} \right)^{T-t} \mathcal{U}(C_{i,T}, C_{T-1}) \mathcal{V}\left( \tilde{h}_{i,T}(\tau), L_{T-1}^{i}\right) W(G_{i,T}, G_{T-1})$$

$$\left[ \mathcal{U}'(C_{i,T}, C_{T-1}) \mathcal{V}'\left( \tilde{h}_{i,T}(\tau), L_{T-1}^{i}\right) + \frac{\theta_{iw}^{T-1} - 1}{\theta_{iw}} \tilde{\sigma}_{iw} \tilde{T}_{T} \Gamma_{T}^{i,j} CPI_{T}(1 + \nu_{T}^{i,j}) \right]$$

where one may recognize the stochastic discount factor between time $t$ and $T$ and between brackets, the wedge between the ratio of the marginal utility of labour and consumption and the real wage with a term in $\theta_{iw}$ representing the market power of households.

Note that this wage setting equation is at the individual level and therefore that the associated utility function and wages depend on the individual consumption of household $\tau$. However, as for the Euler and investment equations, we make the standard assumption that individual dispersion can be

---

\(^8\)These indexations are necessary to ensure that the distribution of wages does not diverge when there is non zero inflation and exogenous growth at steady state.
neglected (see Appendix D).

Although we can describe how wages and labour supply of resetters differ from other households, we can not do so for consumption. With non separable utility, we are thus forced to assume that non resetters and resetters have similar consumption within each type.\(^9\)

The rest of the calculus (steady state and linearisation) is detailed in the subsequent sections, but because the consumption of the Ricardian and non Ricardian household differ, there will be two symmetric Phillips curves for two different wages.

2.2.5 Households’ type aggregation

The introduction of two types of households results in additional aggregation rules for consumption, labour and wages. As seen previously, the existence of two different Phillips curves for Ricardian and non Ricardian households implies that their consumption, labour and wage patterns differ.

In terms of consumption, we now have for consumption of good \(j\) in country \(i\) and total consumption:

\[
C_{j,t}^i = C_{NR,j,t}^i + C_{R,j,t}^i
\]

\[
C_t^i = C_{NR,t}^i + C_t^R
\]  

(2.36, 2.37)

where \(C_{R,j,t}^i\) and \(C_{NR,j,t}^i\) respectively denote consumption of Ricardian and non Ricardian households of good \(j\) in country \(i\). Consumption of the two types of households is simply additive: both types consume the same goods at the same prices with the same imported and domestic share.

Labour and wages are not directly additive, but payroll is by construction. Similarly to labour and wage in both countries we define aggregate labours and wages for both types of households:

\[
L_{NR,t}^i = (\mu^i n^i N) \frac{1}{1-\theta^w} \left( \int_{NR,i} l_i(\tau) \frac{\partial \pi_{w-1}}{\partial \pi_w} \, d\tau \right) \frac{\partial \pi_w}{\partial \pi_{w-1}}
\]

(2.38)

\[
L_{R,t}^i = ((1 - \mu^i n^i N) \frac{1}{1-\theta^w} \left( \int_{R,i} l_i(\tau) \frac{\partial \pi_{w-1}}{\partial \pi_w} \, d\tau \right) \frac{\partial \pi_w}{\partial \pi_{w-1}}
\]

(2.39)

\(^9\)Note that these consumptions converge to the same steady state and follow the same dynamic equations (respectively the Ricardian Euler equation for consumption and the non Ricardian budget constraint).
The corresponding wages are:

\[
W_{i}^{NR,j} = (\mu^{i}n^{j}N)^{\frac{1}{\theta_{w}-1}} \left( \int_{t}^{N_{R,j}} w_{i}^{j}(t)^{1-\theta_{w}} d\tau \right)^{\frac{1}{1-\theta_{w}}},
\]

\[
W_{i}^{R,j} = ((1 - \mu^{i})n^{j}N)^{\frac{1}{\theta_{w}-1}} \left( \int_{R,i} w_{i}^{j}(t)^{1-\theta_{w}} d\tau \right)^{\frac{1}{1-\theta_{w}}}.\tag{2.40}
\]

Labour aggregation read as follows:

\[
L_{i}^{1} = (nN)^{\frac{1}{\theta_{w}-1}} \left( \int_{0}^{nN} l_{i}(\tau)^{\frac{1}{\theta_{w}}-1} d\tau \right)^{\frac{1}{\theta_{w}-1}} = (nN)^{\frac{1}{\theta_{w}-1}} \left( \int_{N_{R,1}}^{R_{1}} l_{i}(\tau)^{\frac{1}{\theta_{w}}-1} d\tau + \int_{R,1}^{NR,1} l_{i}(\tau)^{\frac{1}{\theta_{w}}-1} d\tau \right)^{\frac{1}{\theta_{w}-1}}
\]

\[
= \left( \mu^{i} \left( L_{i}^{NR,1} \right)^{\frac{1}{\theta_{w}}-1} \right)^{\frac{1}{\theta_{w}}} + (1 - \mu^{i}) \left( L_{i}^{R,1} \right)^{\frac{1}{\theta_{w}}-1}. \tag{2.42}
\]

\[
L_{i}^{2} = \left( \mu^{i} \left( L_{i}^{NR,2} \right)^{\frac{1}{\theta_{w}}-1} \right)^{\frac{1}{\theta_{w}}} + (1 - \mu^{i}) \left( L_{i}^{R,2} \right)^{\frac{1}{\theta_{w}}-1}. \tag{2.43}
\]

Wage aggregation reads:

\[
W_{i}^{j} = \mu^{i} \left( W_{i}^{NR,j} \right)^{1-\theta_{w}} + (1 - \mu^{i}) \left( W_{i}^{R,j} \right)^{1-\theta_{w}} \tag{2.44}
\]

In real terms, using the variable used in the linearised model, this also rewrites:

\[
\left( RW_{i}^{j} \right)^{1-\theta_{w}} = \mu^{i} \left( RW_{i}^{NR,j} \right)^{1-\theta_{w}} + (1 - \mu^{i}) \left( RW_{i}^{R,j} \right)^{1-\theta_{w}} \tag{2.45}
\]

with \( RW \) the purchasing power of net wages that is deflated by the country’s VAT included CPI.

### 2.3 Firms

We assume an exogenous and global technological growth process in the form \( \zeta_{i}^{j} = \hat{e}_{i}^{j}(1 + g)^{l} \), where \( g \) is the deterministic growth rate of total factor productivity, \( \hat{e} \) the de-trended steady state level of technology, and \( \hat{e}^{l} \) a possibly autocorrelated stochastic productivity shock. In the rest of the paper, we denote \( Tr_{j} = (1 + g)^{l} \) the deterministic trend of TFP. We assume that technology can be shared and transferred within the union, so that TFP growth is the same in both countries.\(^{10}\)

\(^{10}\)However, the steady state detrended level of TFP, \( \zeta' \), differs across countries to take into account the initial differences in wealth across countries.
2.3.1 Production factors optimisation

Firms hire domestic labour at the cost \( W_i(1 + \nu_{i}^{m,j}) \), where \( \nu_{i}^{m,j} \) is the payroll tax rate levied by the government on firms.\(^{11}\)

Firms also rent capital from households at rate \( r_{i}^{k,j} \). In real term the rental cost of demanded capital \( K_{t}^{d,j}(\varepsilon) \) is then \( r_{i}^{k,j} K_{t}^{d,j}(\varepsilon) \) paid at time \( t \). In nominal terms, this cost equals \( r_{i}^{k,j} K_{t}^{d,j}(\varepsilon) CPI_{t} \): the value of the rented capital in current \( \varepsilon \) is equal to the real capital stock times its market price \( CPI_{t} \).\(^{12}\) Note that capital from previous period is used for production at time assuming installation delays. Therefore at market equilibrium, we have on aggregate \( K_{t}^{d,j} = K_{t-1}^{d,j} \).

In each country \( i \), firm \( \varepsilon \) produces the differentiated good \( y_i^{d}(\varepsilon) \) with the following technology:

\[
y_i^{d}(\varepsilon, t) = \left( \frac{c_{i}^{d,j}}{L_{i}^{d,j}(\varepsilon)} \right)^{1-\alpha} \left( K_{t}^{d,j}(\varepsilon) \right)^{\alpha} \quad \text{at cost} \quad W_i(1 + \nu_{i}^{m,j})L_i^{d,j}(\varepsilon) + r_{i}^{k,j} CPI_{t} K_{t}^{d,j}(\varepsilon),
\]

where \( \alpha \) is the share of capital costs in value added. For sake of simplicity, although it may prove empirically relevant (Challe and Ragot), we do not assume that Ricardian and non Ricardian households correspond to different types of workers. Firms hire both types of households indistinctly.

Every period, firms can reset the quantity of each production factor they use taking wage, taxes and rental cost as exogenous. The arbitrage condition between labour and capital demand yields:

\[
\frac{1 - \alpha}{\alpha} = \frac{W_i(1 + \nu_{i}^{m,j})L_i^{d,j}(\varepsilon)}{r_{i}^{k,j} K_{t}^{d,j}(\varepsilon) CPI_{t}} \quad \text{on aggregate} \quad \frac{1 - \alpha}{\alpha} = \frac{W_i(1 + \nu_{i}^{m,j})L_i^{d,j}(\varepsilon)}{r_{i}^{k,j} K_{t-1}^{d,j} CPI_{t}}.
\]

The marginal cost of production is identical across firms and does not depend on its size:

\[
MC_{i}^{j}(\varepsilon) = MC_{i}^{d,j} = \frac{1}{\alpha(1 - \alpha)^{1-\alpha}} \left( \frac{W_i}{c_{i}}(1 + \nu_{i}^{m,j}) \right)^{1-\alpha} \left( r_{i}^{k,j} CPI_{t} \right)^{\alpha}
\]

\[
RMC_{i}^{j} = \frac{MC_{i}^{j}}{P_{i}^{d,j}} = \frac{1}{\alpha(1 - \alpha)^{1-\alpha}} \left( \frac{RW_{i}^{j}}{c_{i}}(1 + \nu_{i}^{m,j})(1 + \nu_{i}^{m,j}) \right)^{1-\alpha} \left( \frac{r_{i}^{k,j}}{P_{i}^{d,j}} \right)^{\alpha}
\]

2.3.2 Price setting

The price setting follows Calvo process in each country. Firm \( \varepsilon \) can reset its price with exogenous probability \( (1 - \xi_{i}) \). Producers know the relationship between their price and the demand for their product

\(^{11}\) No taxes on labour income (social contribution, income tax) are paid by households here. The steady state is not affected by this assumption but the reaction of wages to this tax is affected in the short-term.

\(^{12}\) The price of capital is by convention the same as the price of investment, which is identical to the price of consumption as we assume that both goods are identical.
and choose their price $\tilde{P}_i^T(\epsilon)$ so as to maximise their expected profit under that constraint:

$$\max_{\tilde{P}_i^T(\epsilon)} \sum_{t=1}^{\infty} (P_t^i L_t^i)^{-\gamma_p} \left( \frac{\tilde{P}_i^T(\epsilon)}{P_t^i} \right)^{-\theta_p} \Pi_t^i, \quad \text{(2.50)}$$

subject to

$$y_{i,T}^i(\epsilon) = \left( \frac{1}{\alpha} \right) \left( \frac{P_t^i}{\gamma_p} \right)^{-\alpha} \left( K_t^p(\epsilon) \right)^{1-a}, \quad \text{(2.51)}$$

$$1 - \alpha = \frac{W_t^i(1 + \nu_t^i) L_t^i(\epsilon)}{r_t^i K_t^d(t) CPI_t^i}, \quad \text{(2.52)}$$

$$\Pi_t^i(\epsilon) = \tilde{P}_i^T(\epsilon) \prod_{k=1}^{T-1} (\Pi_t^k)^{\gamma_p} (\Pi_t^{i-1})^{1-\gamma_p} = \tilde{P}_i^T(\epsilon) \Gamma_t^{T-1}, \quad \text{(2.53)}$$

where the Lagrange multiplier $\lambda_t^i$ is the marginal utility of a representative *Ricardian* households in country $i$. $\gamma_p$ is the parameter of price indexation on past inflation and $\Gamma_t^{T-1}$ denotes $\prod_{k=1}^{T-1} \Pi_t^{k+1} (\Pi_t^{i-1})^{1-\gamma_p}$. So $\Pi_t^i(\epsilon) = \tilde{P}_i^T(\epsilon) \Gamma_t^{T-1}$ is the price of good $i$ of country $i$ at time $T$ when its price was last reset at time $t$. $\gamma_p$ is the parameter of price indexation on past inflation and $\Gamma_t^{T-1}$ denotes $\prod_{k=1}^{T-1} \Pi_t^{k+1} (\Pi_t^{i-1})^{1-\gamma_p}$. So $\Pi_t^i(\epsilon) = \tilde{P}_i^T(\epsilon) \Gamma_t^{T-1}$ is the price of good $i$ of country $i$ at time $T$ when its price was last reset at time $t$. Note that $\Pi_t^i$ is the inflation of goods produced in country $i$ and differs from inflation of the consumption price index $CPI_t^i$, which includes inflation from imported goods as well. $\Pi_t^1$ is the steady state value of $\Pi_t^1$.

The first order condition reads:

$$0 = \sum_{t=1}^{\infty} (P_t^i L_t^i)^{-\gamma_p} \left( \frac{\tilde{P}_i^T(\epsilon) \Gamma_t^{T-1}}{P_t^i} \right)^{-\theta_p} \left( \frac{\tilde{P}_i^T(\epsilon) \Gamma_t^{T-1} - \theta_p}{\gamma_p - 1} M_t^{C_t} \right) \quad \text{(2.55)}$$

### 2.3.3 Dividends distribution

Firms cannot save or invest, so they redistribute their profits to households. This distribution can be thought of as dividends to firms owners, if negative it is similar to a recapitalisation of the firm. Therefore, we assume that only unconstrained households, who have access to financial and investment markets, are paid such dividends $D_i^t$.

$$D_i^t = \tilde{P}_t^i Y_i^t - W_t^i (1 + \nu_t^i) L_t^i - r_t^j K_{t-1}^d CPI_t^i \quad \text{(2.56)}$$

$$D_i^t = \tilde{P}_t^i Y_i^t (1 - RMCI) \quad \text{(2.57)}$$

---

\[13\] These households own the firms, so logically their utility enters the price-setting program. This is however neutral on the linearised Phillips curve apart from a redefinition of $\beta$ when there is long term growth, a redefinition which does not depend on households type.
2.3.4 Aggregate production function

We assume that the production function is identical across firms. We can compute the aggregated production function based on the definition of $Y_i^t$ as a function of $y^i_t(\varepsilon)$:

\[
Y_i^t = \left( p^i \frac{1}{p^P} \right)^{\frac{\theta}{\theta - 1}} \left( \int_0^{p^P} y^i_t(\varepsilon) \frac{d^{\theta - 1}}{d\varepsilon^{\theta - 1}} d\varepsilon \right)^{\frac{\theta}{\theta - 1}} = \left( \zeta_i^t L_i^t \right)^{1-\alpha} \left( K_i^{d_j} \right)^{a} \left( p^i \frac{1}{p^P} \right)^{\frac{\theta}{\theta - 1}} \left( \int_0^{p^P} \left( \frac{L_j^t(\varepsilon)}{K_j^{d_j}} \right)^{1-\alpha} \frac{d^{\theta - 1}}{d\varepsilon^{\theta - 1}} d\varepsilon \right)^{\frac{\theta}{\theta - 1}} \Delta_i^t
\]

which simplifies because of equation (2.47):

\[
= \left( \zeta_i^t L_i^t \right)^{1-\alpha} \left( K_i^{d_j} \right)^{a} \left( p^i \frac{1}{p^P} \right)^{\frac{\theta}{\theta - 1}} \left( \int_0^{p^P} \left( \frac{L_j^t(\varepsilon)}{K_j^{d_j}} \right)^{1-\alpha} \frac{d^{\theta - 1}}{d\varepsilon^{\theta - 1}} d\varepsilon \right)^{\frac{\theta}{\theta - 1}} = \left( \zeta_i^t L_i^t \right)^{1-\alpha} \left( K_i^{d_j} \right)^{a} \Delta_i^t
\]

It follows that the production function on aggregate includes a measure of firm size dispersion. The productivity shock could be assumed to encompass both aspects: individual productivity and size dispersion; however the pure productivity shock appears on its own in the Phillips curve on prices (through the marginal cost of production). Hence, following the common implicit assumption in this literature, we assume that dispersion is stable enough for $\Delta_i^t$ to be taken as constant, assumption verified at first order (see Appendix D). Moreover at the steady-state, firms are all identical, that is of same size $\frac{1}{p^P}$. It follows that the steady state value of the dispersion index $\Delta$ is $\bar{\Delta}^i = 1$.

2.4 Fiscal Authorities

By and large, the purpose of governments is to stimulate domestic production, labour and individual consumption, as well as to provide with public and collective goods and services.

In the real world, fiscal policy is implemented through a large number of instruments such as public expenditures, investment, or employment, as well as through the taxation and social contribution system. However, this large number of instruments cannot be modelled in details and one need to resort to simplifications.

**Tax system** First, in MELEZE, we assume that tax rates over consumption, labour and capital incomes are exogenous and are discretely chosen by governments. This choice is consistent with a low variability of apparent tax rates in the data over the calibration period.
Transfers  Second, in the present model, governments can raise lump-sum taxes or commit to lump-sum transfers to households. In the baseline behaviour of MELEZE, these transfers are held constant. However, we allow for potential exogenous transfers shocks, targeted on Ricardian or non Ricardian households.

Here, nominal lump-sum transfers from the government are additive $\Phi_i^t = \Phi_i^{NR,t} + \Phi_i^{R,t}$, and we chose the most simple way to distribute transfers (default setting), that is in proportion to their population share: $\Phi_i^{NR,t} = \mu_i \Phi_i^t$ and $\Phi_i^{R,t} = (1 - \mu_i) \Phi_i^t$.

These transfers can easily be endogenised to reflect the redistributive policy of the welfare state within the cycle. At steady state, any redistribution weights can be rationalized by the relative weights assigned by the government to both types of households.

Public expenditures  In the absence of public production or employment in the present model, we capture and encompass all remaining dimensions of public intervention through public expenditures.

In order to model fiscal policy, these expenditures are decomposed between an endogenous component answering to economic developments and an exogenous and discretionary component. In standard DSGE models, the endogenous behaviour is either modelled as the solution to a Ramsey optimality problem or through budget rules estimated ex ante. In the following sections, we take a closer look at different budget rules and present an alternative to the Ramsey approach to model public expenditures.

Moreover, and to model the persistence of government expenditures (the welfare state cannot be dramatically reshaped overnight), we introduce habits on public consumption in the households’ utility function.

2.4.1 Budget and fiscal rules

Budget rules can be implemented in different ways all relying on the ad hoc description of governments’ expenditures as a function of observable endogenous variables.

1. Exogenous public spending  In closed economy when fiscal policy is not the purpose of the model, the government behaviour can be greatly simplified: one can assume (i) the absence of taxes, (ii) exogenous public expenditure, and (iii) public debt as a mere counterpart of private assets. In this specific case, lump-sum transfers are assumed to balance the government budget constraint. For instance, Negro, Schorfheide, Smets, and Wouters (2007) or Justiniano and Primiceri (2008)
consider a budget rule of the following form:

\[ G_i^t = \left( 1 - \frac{1}{g_i^t} \right) Y_i^t \]  

(2.60)

where \( g_i^t \) is an exogenous disturbance following the process:

\[ \log g_i^t = (1 - \rho g_i^t) \log g_i^{t-1} + \sigma_y^i \epsilon_{g,t} \]  

(2.61)

However, such a rule can not be incorporated in our open economy model. As there is explicit public debt, there must be some mechanism ensuring the existence of a steady state for public expenditure and debt altogether in both countries.

2. **Endogenous spending without distortive taxes** Corsetti, Meier, and Müller (2010), in a monetary union model, consider three fiscal instruments, namely spending, transfers and debt modelled as follows:

\[ G_i^t = (1 - \Psi_{gg}) G_i^{t-1} + \Psi_{gy}(Y_i^{t-1} - Y_i^t) + \Psi_{gd} \frac{PA_i^t}{P_i^t} + \epsilon_{g,t} \]  

(2.62)

\[ \frac{\Phi_i^t}{P_i^t} = G_i^t \left( \frac{P_i^t G_i^t}{\bar{P}^t G_i^t} \right) + \Psi_{gd} \frac{PA_i^t}{\bar{P}^t} \]  

(2.63)

where \( Y_i^{t-1} \) denotes the level of output that would prevail under flexible prices and wages. At steady state with these specifications, there is no public debt. Public spending may be made exogenous \((\Psi_{gy} = \Psi_{gd} = 0)\), in which case transfers adjust automatically to ensure the convergence of public debt to its steady state. Corsetti et al. (2010) compare the previous case to a situation where public spending is adjusted endogenously to contribute to the convergence of public debt and may in addition be pro or contra cyclical depending on the value of \( \Psi_{gy} \).

3. **Endogenous spending with distortive taxes** In models where fiscal policy is more detailed, adjustment can be made through tax rates. For instance, Carton and Guyon (2012) consider that the VAT rate follows a fiscal rule which enables their monetary union model to reach a steady state:

\[ v_i^c - \nu_i^c = \rho_G (v_i^c - \nu_i^c) + (1 - \rho_G) \left[ \beta_{BG}(pa_i^t - \bar{pa}_i^t) + \beta_{DG}(pa_i^t - \bar{pa}_i^{t-1}) \right] \]  

(2.64)

Under this form, the VAT rate will gradually adjust to equalize the public debt to its targeted level.

---

[14] Other instruments available are social contributions, transfers or public expenditures.
In the European Commission model QUEST III, Ratto, Roeger, and in’t Veld (2009) consider a much richer fiscal policy. It is assumed that the government reacts to its own measure of the output gap, which differs from the wedge between output under staggered and flexible prices usually considered for monetary policy analysis. This variable influences government spending, public investment and the income tax rate. Pensions and unemployment benefit react to the population structure and aggregate wages. A lump-sum tax ensures the convergence of public debt to a targeted level at steady state by reacting to both debt and deficit.

Modelling governments’ behaviour through fiscal rules is a flexible approach, however it is subject to two issues. First, budget rules postulate a behaviour rather than an objective for the government and are therefore subject to the Lucas critique as they do not identify structural parameters for the government’s behaviour. Second, when designing a budget rule, one should be particularly cautious as rules may not be compatible with the existence and uniqueness of the model’s solution if long-term solvency of the government is not properly ensured.

2.4.2 Optimizing government: a simplified approach to the Ramsey problem

Rationale  The introduction of rationality in DSGE models historically and naturally lead to the definition of an optimal government behaviour as a normative benchmark, namely the Ramsey policy. Indeed, in a internally consistent DSGE approach, governments seek to maximize the welfare of their domestic households, and it is therefore natural to define the objective of fiscal authorities as the maximization of the intertemporal utility of households. In the presence of rationality, this maximisation is indeed subject to the public budget constraint but also to the full set of model constraints. In particular, when choosing the optimal level of public expenditures $G_t$, the government internalizes its indirect impact on households’ consumption and labour supply, and therefore households’ utility.

One strength of this standard Ramsey approach is its robustness to the Lucas critique as it defines a structural behaviour consistent with the hypotheses of the model. In addition, as we introduce government spending in the utility function in MELEZE, this Ramsey approach appears to be even more strongly justified.

However, solving a Ramsey problem is both analytically and numerically complex (when not infeasible) in large models, especially within the business cycle, as well as unrealistic as it does not embody political choices observed in the real world that may depart from optimality. This reason underlies the classical choice of ad hoc budget rules in DSGE models.

As an alternative to these rules, we propose a new approach based on a simplified version of the Ramsey problem where the government still maximizes households’ utility subject to its transfers/tax
revenues budget constraint, however not taking into account all other constraints. Concretely, the government solves the Ramsey problem taking endogenous variables other than public expenditures as given (such as \( C_i^T(\tau) \) and \( L_i^T(\tau) \) here)\(^{15}\). As a result, such a government focuses only on the utility derived by households through the direct action of the government rather than through second turn effects on consumption and labour. As for budget rules, this remains inconsistent with the DSGE approach of a full knowledge of economic mechanisms by agents. However, this may also be interpreted as a difficulty for fiscal authorities to exactly assess the impact of its policies on the economy.

Closer to the full Ramsey problem, we believe this approach to be more robust to the Lucas critique than traditional budget rules as it partially micro-founds the behaviour of the government.

However, both approaches suffer from the same paradoxes when embedded in a general equilibrium model solved under rational expectations. First, in order to solve for such a model, expectations of all agents are assumed formed through the entire model. It is then paradoxical to assume that either the government maximizes its objective under a subset of constraints or maximizes an implicit objective through a rule defined outside the model. Second, both modelling are only simple descriptions of fiscal authorities and do not encompass real-world phenomena such as the will of authorities to get reelected that may induce sub-optimal behaviours.\(^{16}\)

**Program and objective of the government** As the government now seeks to maximise the intertemporal flow of utility across all households \( \tau \), defining weights for each households (ie. the cumulative distribution function \( F_\tau \)), the government’s objective \( O_t^G \) is given by the aggregation of these intertemporal flows:\(^{17}\)

\[
O_t^G = \int_{\tau \in i} \left\{ E_t \sum_{T=t}^{\infty} \beta_i^T - T \left\{ U(C_i^T(\tau),C_i^{T-1}) V(L_i^T(\tau),L_i^{T-1}) W(G_i^T,G_i^{T-1}) \right\} \right\} d F(\tau)
\]

\[
= E_t \sum_{T=t}^{\infty} \beta_i^T - T W(G_i^T,G_i^{T-1}) \int_{\tau \in i} \left\{ U(C_i^T(\tau),C_i^{T-1}) V(L_i^T(\tau),L_i^{T-1}) \right\} d F(\tau)
\]

\[
= E_t \sum_{T=t}^{\infty} \beta_i^T - T W(G_i^T,G_i^{T-1}) \Omega_T
\]

Under the reasonable assumption that the government cannot distinguish households within the same sub-group\(^{18}\), denoting \( \omega^{R,j}_k \) the weight on Ricardian agents, and neglecting the intra-group dispersion in

\(^{15}\)This choice of modelling is equivalent to consider that the government behaves under bounded rationality.

\(^{16}\)See for instance, the public choice theory literature.

\(^{17}\)With separable utility, the government can restrict to maximize the intertemporal flow of utility \( W^{sep}(G_i^T,G_i^{T-1}) \) alone. \( U(C_i^T(\tau),C_i^{T-1}) \) and \( V(L_i^T(\tau),L_i^{T-1}) \) terms disappear.

\(^{18}\)That is for instance, the government cannot distinguish and weight differently two Ricardian households. However, it can put a different weight on Ricardian and non Ricardian agents.
labour and consumption, the weighting factor $\Omega_i$ writes:

$$
\Omega_T = n^i N \omega^{R,i} U \left( \frac{C^R_i}{(1 - \mu^i)n^iN} C_{T-1}^R, \frac{L^R_i}{(1 - \mu^i)n^iN}, L_{T-1}^i \right) 
+ (1 - \omega^{R,i}) n^i N \omega^{NR,i} U \left( \frac{C^{NR,i}}{\mu^i n^i N}, C_{T-1}^{NR} \right) \left( \frac{L^{NR,i}}{\mu^i n^i N}, L_{T-1}^{NR} \right) 
$$

(2.66)

All in all, in the most general case, the government’s program is as follows:

$$
\max_{G_i, PA_i} E_T \sum_{T=1}^\infty \beta^T W(G_T, G_{T-1}) \Omega_T(C^R_T, C^{NR}_T, C_{T-1}, L^R_T, L^{NR}_T, L_{T-1}) 
$$

(2.67)

with $W(G_T, G_{T-1}) = \left( G_T \left( C_{T-1} \right) \right)^{-k^i} \eta(1-c^i)$

(2.68)

s.t. $PA_i = \left( R_{t-1} - \psi (PA_{t-1}^{i-1}) \right)^{\psi} \left( \frac{PA_{t-1}^i}{P_{t-1}^{i-1} Y^{T Tr_{t-1}}} \right)$ $PA_{t-1}^i + \psi^{\psi} W_i L_i^{i} + \psi^{\psi} r_i^{i} CPI_i K_i^{i}$

(2.69)

$$
+ \psi^{\psi} CPI_i^i (C_i + I_i^i) + \psi^{\psi} D^i_i + \psi^{\psi} FD^i_i - P_i^i G_i^i - \Phi_i^i 
$$

where $PA_i^i$ denotes the nominal public assets of country $i$ at the end of period $t$, and $\Phi_i^i$ are nominal transfers to households.

Note that the real interest rate for governments differs from that of households because their consumptions are priced differently, governments buying exclusively domestic production. Also the atomicty hypothesis made for households relative to the asset market does not hold for governments and their debt premia differ ($\psi$ versus $\psi^\psi$). Besides, the discount factor of the government needs not be equal to that of households. On the one hand, the government, as an institution, is longer lived than its citizens and for this reason could put a higher weight on future utility than households do. On the other hand, as political entities aimed at satisfying voters and winning elections, governments may also put a higher weight on the near future.

Solving for the previous program yields the following Euler equation for government consumption that defines the default behaviour of fiscal authorities in MELEZE:

$$
E_t f^{i} \frac{W_i(G_{t+1}, G_t) \Omega_{t+1} + \beta^i E_t + W_2(G_{t+1}, G_t) \Omega_{t+1}}{W_i(G_{t+1}, G_t) \Omega_t + \beta^i E_t W_2(G_{t+1}, G_t) \Omega_{t+1}} - \psi \frac{PA_i}{\Pi^{Y \ Y Tr_t}} - \psi^\psi \frac{PA_i}{\Pi^{Y \ Y Tr_t}} = 1 
$$

(2.70)
2.5 Financial Intermediation

The stationarity of an open economy model is not straightforward. As explained by Schmitt-Grohé and Uribe (2003), in a small open economy model, it can be ensured by some modelling elements, which are usually not microfounded. The literature on monetary union model usually borrows the same solutions. In our model, we microfound one of Schmitt-Grohé and Uribe’s proposals (debt elastic spreads) and introduce a simplified international financial market.

We assume that there exists an international financial market for assets (private or public). On the financial market, intermediaries can borrow money from the central bank of the monetary union to finance public or private credit, and conversely borrow money from agents to deposit it at the central bank. Through financial intermediaries, private (resp. public) agents can borrow or lend money by paying a debt premia $\psi$ (resp. $\psi^g$). The interest rate for the exchange between the central bank and the financial intermediary is the interest rate set by the central bank.

We assume that financial intermediaries work in perfectly competitive market. They operate a war on prices (spreads), up to a point where they make no profit. To ensure the orthogonality of financial intermediaries with respect to the rest of the monetary union, we assume that their unique cost is the refinancing cost vis-à-vis the central bank. Assuming so generates no wage payment or capital and intermediate consumption purchases in this branch of activity hence no transfer between the real economy within the monetary union and financial operators located outside this union. Therefore developments on the financial market do not affect the rest of the system. The optimisation program of financial intermediaries is not needed to close our model. One could for instance assume that financial activities are based strictly out of the monetary union, for instance in England or in Switzerland.

As a result, if households or the government in country $i$ are net borrowers (i.e. $FA_i^t \leq 0$ or $PA_i^t \leq 0$), this agent has to pay an interest premium on his debt amounting to $|\psi(fa_i^t)|$, $|\psi^g(pa_i^t)|$. When the agent is net lender, returns are reduced by this same spread captured by the intermediary. This mechanism is equivalent to financial intermediation services (FISIM, see Figure 1). In our model, there is no explicit risk or asymmetry of information so that the financial intermediation comes down to the spread between the refinancing rate offered by the Central Bank and the market rate set by the commercial bank without risk, term or other premium.

Concretely, the aggregate cash needs financial intermediaries borrow from the central bank are the opposite of all agents asset holdings:

$$CN_i = -(FA_i^1 + FA_i^2 + PA_i^1 + PA_i^2)$$  \hspace{1cm} (2.71)
Figure 1: Debt elastic spreads are similar to FISIM

The production of financial intermediation services associated is given by the amount of spread paid today by agents on their stock of financial assets available at the end of previous period:

$$FY_t = \sum_{i=1,2} \psi \left( \frac{FA^i_{t-1}}{P^i_{t-1} Y^i T_r_{t-1}} \right) FA^i_{t-1} + \sum_{i=1,2} \psi g \left( \frac{PA^i_{t-1}}{P^i_{t-1} Y^i T_r_{t-1}} \right) PA^i_{t-1}$$

(2.72)

As for good producing firms, we assume that financial intermediaries are owned by Ricardian households. However, ownership is transnational. We do not introduce in the model any labour or capital for the financial intermediation industry and simply assume that all benefits are paid lump-sum to Ricardian households ($FD_{1,2}^i$). Moreover, these benefits are paid in proportion $\theta^f a$ (c.f. section A.9).

$$FD_1^i = \frac{\theta^f a}{1 + \theta^f a} FY_t$$

(2.73)

$$FD_2^i = \frac{1}{1 + \theta^f a} FY_t$$

(2.74)

$$FD_1^i + FD_2^i = \sum_{i=1,2} \psi \left( \frac{FA^i_{t-1}}{P^i_{t-1} Y^i T_r_{t-1}} \right) FA^i_{t-1} + \sum_{i=1,2} \psi g \left( \frac{PA^i_{t-1}}{P^i_{t-1} Y^i T_r_{t-1}} \right) PA^i_{t-1}$$

(2.75)

In real terms, this rewrites:

$$\frac{FY_t}{P^i_r Y^i T_r} = \sum_{k=1,2} \psi \left( \frac{FA^k_{t-1}}{P^k_{t-1} Y^k T_r_{t-1}} \right) FA^k_{t-1} + \sum_{k=1,2} \psi g \left( \frac{PA^k_{t-1}}{P^k_{t-1} Y^k T_r_{t-1}} \right) PA^k_{t-1}$$

(2.76)
Denoting $f_d^i = \frac{FD^i}{\bar{Y}^i Y_t}$, we have:

$$f_d^1 = \frac{\theta f_a}{1 + \theta f_a} \frac{FY_t}{\bar{Y}^1 Y_t}$$

$$= \frac{-\theta f_a}{1 + \theta f_a} \left( \psi(f_a^1_{t-1}) f_a^1_{t-1} + \psi^s(pa^1_{t-1}) pa^1_{t-1} \right) + \frac{T_{t-1}}{\theta} \left[ \psi(f_a^2_{t-1}) f_a^2_{t-1} + \psi^s(pa^2_{t-1}) pa^2_{t-1} \right] \frac{1}{\Pi^1_t} \frac{Tr_{t-1}}{Tr_t} \tag{2.77}$$

$$f_d^2 = \frac{1}{1 + \theta f_a} \frac{FY_t}{\bar{Y}^2 Y_t}$$

$$= \frac{1}{1 + \theta f_a} \left( \psi(f_a^2_{t-1}) f_a^2_{t-1} + \psi^s(pa^2_{t-1}) pa^2_{t-1} \right) + \frac{\theta}{T_{t-1}} \left[ \psi(f_a^1_{t-1}) f_a^1_{t-1} + \psi^s(pa^1_{t-1}) pa^1_{t-1} \right] \frac{1}{\Pi^2_t} \frac{Tr_{t-1}}{Tr_t} \tag{2.78}$$

Also, to mimic the interbank overnight markets where banks clear their daily position towards the central bank by lending or borrowing according to the refinancing rate, we assume that at each period, the financial intermediaries clear their position towards the central bank:

$$CN_t = -(FA^1_t + FA^2_t + PA^1_t + PA^2_t) = 0 \tag{2.79}$$

This zero cash needs condition can also be read as public debt being held entirely by households within the union. In section E we show that assuming no aggregate debt in the monetary union at steady state implies this zero cash needs constraint at all dates. This constraint ensures that the model satisfies the Walras law, i.e. that once all markets are cleared, three out of four laws of motion of assets (public and private in both countries) imply the fourth one. The economic property of being Walrassian implies that the steady state is stable and the solution to the linearised model is unique.

### 2.6 Monetary Authority, Prices and Inflation

The central bank sets the nominal interest rate $R_t$ common to both countries through a Taylor rule (Taylor, 1993), where it reacts to both current inflation of the consumption price index and to the output gap.

$$R_t = R^\rho_{t-1} \left( R^* \left( \frac{\Pi^\text{union,VAT}_t}{\Pi^*} \right)^{\rho_\pi} \bar{Y}_t^{\gamma_\pi} \right)^{1-\rho} \tag{2.80}$$

where $\Pi^\text{union,VAT}_t$ and $\bar{Y}_t$ are respectively the VAT-included average inflation of consumption in the monetary union, and the total output gap of the monetary union (see Appendix C.1). $R^*$ is the interest-rate
target of the central bank and $\Pi^*$ its exogenous inflation target. $r_\pi$ and $r_y$ are the Taylor rule weights assigned to inflation and the output gap, $\rho$ is the interest-smoothing parameter.

As there is no union-wide maximizing households embedded in the model, union aggregate price index and aggregate output gap cannot be directly inferred. A way to bypass this issues can be to assume that for instance, the aggregate price index is consumption weighted geometric average of the national price indexes as in Eggertsson, Ferrero, and Raffo (2014).

In our model, we choose to derive approximation of the national-accounting exact definitions of both the aggregate price index and the aggregate output gap for the monetary union. The derivations are presented in Appendix C.1 and allow to define:

$$\Pi_{\text{union}, \text{VAT}} = \frac{1}{1 + \frac{1 + \nu_1}{\text{CPI}_1} + \frac{1 + \nu_2}{\text{CPI}_2}}, \quad \Pi_{\text{VAT}}^{\text{C},1}$$

and

$$\Pi_{\text{VAT}}^{\text{C},2} = \frac{1}{1 + \frac{1 + \nu_1}{\text{CPI}_1} + \frac{1 + \nu_2}{\text{CPI}_2}},$$

where $\Pi_{\text{union}, \text{VAT}}$ refers to the VAT-included CPI inflation.

### 2.7 Market Clearing and relative prices

Every period, markets clear in quantities in both countries:

$$Y^i_t = C^i_{t,j} + C^j_{t,i} + I^i_{t,j} + I^j_{t,i} + G^i_t.$$

(2.83)

In values, this becomes:

$$P^i_t Y^i_t = P^i_t (C^i_{t,j} + I^i_{t,j}) + P^j_t (C^j_{t,i} + I^j_{t,i}) + P^i_t C^i_t + P^j_t (C^j_{t,i} + I^j_{t,i}) - P^i_t (C^j_{t,i} + I^j_{t,i}),$$

(2.84)

which can also be written under the well-known form:

$$P^i_t Y^i_t = CPI^i_t (C^i_t + I^i_t) + P^i_t C^i_t + P^j_t X^i_t - P^i_t M^i_t,$$

(2.85)

where $X^i_t$ is the exports sold to country $j$ at the price of the domestic good. Likewise, the imports $M^i_t$ are bought from country $j$ at price $P^j_t$. Because demand for foreign goods is addressed by households for consumption and investment, we have $M^i_t = C^j_{t,i} + I^j_{t,i} = X^i_t$.

Moreover, the relative prices of consumption $RPC^i_t = \frac{CPI^i_t}{P^i_t}$ with respect to production (net of taxes) are equal to:

\[ RPC^i_t = \frac{CPI^i_t}{P^i_t} \]
\[ RPC_1^t = \left( \frac{P_2^t}{P_1^t} \right)^{\alpha_1} \quad RPC_2^t = \left( \frac{P_1^t}{P_2^t} \right)^{\alpha_2} \] (2.86, 2.87)

We denote the terms of trade

\[ T_t = \frac{P_2^t}{P_1^t} \] (2.88)

3 Calibration

The model being derived, Appendix A details the steady states relationships between variables. Taking into account all these relationships imposes crucial restrictions on structural parameters, endogenous ratios to GDP, as well as on endogenous variables in level. We calibrate our quarterly model as to match the situation of France within the Eurozone\(^{19}\) over the period 1995-2007, and as to stay coherent with the traditional DSGE literature for structural parameters.

3.1 Classification of parameters and resolution of the steady state

We distinguish three types of parameters: (i) structural parameters, (ii) policy parameters and (iii) endogenous parameters. Indeed, due to the large numbers of steady state restrictions to account for, some parameters cannot be calibrated freely and are actually endogenously determined by the steady state equations. Structural parameters are parameters (technology, preferences, etc.) deemed purely exogenous, accounting for mechanisms outside of the model and not susceptible to change across simulations. Policy parameters corresponds to discretely chosen parameters by fiscal and monetary authorities such as the inflation target and the tax rates. Lastly, endogenous parameters are constrained by the full steady state model and need to be solved for. In all, parameters are sorted as follows:

- **Structural**:  \( n, \ln, \mu^l, \sigma^l, \sigma^r, \eta^i, h^l, h^r, \alpha, \xi_i, \zeta, \theta^i, \gamma^i_p, \gamma^i_c, \gamma^i_l, \gamma^i_g, \alpha, \psi_i, \theta_i, \gamma_i_p, \gamma_i_c, \gamma_i_l, \delta, \beta_i, \beta_i^g, \kappa_i, \alpha_i, \psi_i^g, \psi_i^f, \psi_i^w, \psi_i^r \)

- **Policy**:  \( \nu^c, \nu^w, \nu^k, \nu^d, \Phi_i, \bar{\Pi}, r_y, r_{\Pi} \)

- **Endogenous**:  \( R, p^d, f^d, \bar{g}, c^f, \bar{y}, \bar{\bar{y}}, \bar{T}, \theta, \) and other endogenous steady state values of endogenous variables.

The number of firms \( p, p^d \) are mute throughout the model, the scale of production being defined by the size of populations \( n, \ln \) and productivities \( \xi_i \).

Following this choice, we explicit a sequential method for the resolution of the steady state that minimizes the number of simultaneous equations systems to solve. This resolution is implemented under R and the main guiding lines are as follows:

---

\(^{19}\)11 countries: Belgium, Germany, Ireland, Greece, Spain, Italy, Luxembourg, Netherlands, Austria, Portugal and Finland.
1. First, given the exogeneity of $\tilde{\beta}_i$, $\tilde{\beta}_k$ and $\Pi$, the Euler equations of both the households and the governments (Equations STEADY.1 and STEADY.6) define the debt to output ratios $\bar{p}a_i$ and $\bar{f}a_i$ as functions of the nominal interest rate $\bar{R}$. As such, the zero cash need condition (Equation STEADY.7) can be interpreted as a market clearing condition defining the price of bonds $\bar{R}$ depending on the supply (public debt) and demand (private savings) for financial assets.

2. Second, we isolate a first system of equations composed of national market clearings (Equation STEADY.9), the definitions of the trade balance through both financial flows (Equation STEADY.14) and trade flows (Equation STEADY.13), the zero cash need condition (Equation STEADY.7) and the governments’ budget constraints (Equation STEADY.12). This system allows to compute the nominal interest rate $\bar{R}$, the trade balance to production ratio $\bar{tb}$, government spendings $gy_i$, consumption and investment to production ratios $(cy_i + iy_i)R\bar{P}\bar{C}_i$, as well as the relative size of nominal value added across countries $\theta^* = \theta/\bar{T} = \bar{Y}_1/\bar{Y}_2$.

3. A second system of simultaneous equations corresponds to the determination of the share of (non) Ricardian agents in consumption $s^c_i$ and payroll $s^l_i$ given exogenous fractions of non Ricardian agents $\mu^i$. This system consists of both the non Ricardian households budget constraints (Equation STEADY.3) and the consumption-leisure arbitrages (Equation STEADY.4).

4. At this point, Equation STEADY.16 allows to compute the level of labour supply in both economies. The resolution is closed by computing the level of production in both countries with use of Equation STEADY.17 and the terms of trade $\bar{T} = (\bar{Y}_1/\bar{Y}_2)/\nu^*$.

5. Others endogenous ratios are then merely computed using straightforward combinations of previous variables.

3.2 Data, calibration and inverse inference

Values for structural parameters, when possible, are chosen from the standard literature on DSGE models, these parameters being estimated either by Bayesian methods on macro data or directly on micro data.

Based on a large literature review, Table 2 presents the calibration of structural parameters as well as their sources.

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<table>
<thead>
<tr>
<th></th>
<th>DATA</th>
<th></th>
<th>MELEZE</th>
<th></th>
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<td>EA (12) excl.</td>
<td>FR</td>
<td>EA (12) excl.</td>
<td>FR</td>
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<td>Output in 2000 (GDP)*</td>
<td>5458</td>
<td>1485</td>
<td>5314</td>
<td>1473</td>
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<td>Output in 2000 (VA excl Financial)*</td>
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<td>1278</td>
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<td>1281</td>
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<td>Output per capita average growth rate**</td>
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<td>1,5 %</td>
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<td>25,7</td>
<td>110,3</td>
<td>25,7</td>
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<tr>
<td>Hours worked per week (since 2000)</td>
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<td>34,3</td>
<td>34,6</td>
<td>34,3</td>
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<tr>
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<td>46 %</td>
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<td>Gross wages to VA</td>
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<td>Nominal 3 month Euribor**</td>
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<td>-</td>
<td>4,0 %</td>
<td>4,0 %</td>
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<td>1,6 %</td>
<td>2,0 %</td>
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<td>Private consumption to GDP ratio</td>
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<td>Public consumption to GDP ratio</td>
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<td>Investment to GDP ratio</td>
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<td>GFCF to Capital ratio</td>
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<td>Imports from Euro area partner†</td>
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<td>3 %</td>
<td>12 %</td>
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<td>PPP (GDP, since 2002)</td>
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<td>PPP (CPI, since 2003)</td>
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<td>Public debt</td>
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<td>-37 %</td>
<td>-51 %</td>
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<td>Private assets including firms (S1 excl. S13)**</td>
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<td>41 %</td>
<td>50 %</td>
<td>41 %</td>
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<td>Net financial position (S2)**</td>
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<td>-3 %</td>
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<td>-3 %</td>
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<td>Tax revenue (in GDP)</td>
<td>40 %</td>
<td>44 %</td>
<td>37 %</td>
<td>40 %</td>
</tr>
<tr>
<td>Implicit tax rate on consumption</td>
<td>20 %</td>
<td>20 %</td>
<td>20 %</td>
<td>20 %</td>
</tr>
<tr>
<td>Consumption tax income (in GDP)</td>
<td>11 %</td>
<td>11 %</td>
<td>13 %</td>
<td>13 %</td>
</tr>
<tr>
<td>Implicit tax rate on labour</td>
<td>38 %</td>
<td>39 %</td>
<td>38 %</td>
<td>39 %</td>
</tr>
<tr>
<td>Labour tax income (in GDP)</td>
<td>21 %</td>
<td>22 %</td>
<td>18 %</td>
<td>18 %</td>
</tr>
<tr>
<td>Capital tax income (in GDP)</td>
<td>8 %</td>
<td>10 %</td>
<td>7 %</td>
<td>8 %</td>
</tr>
<tr>
<td>Transfers (in GDP)</td>
<td>16 %</td>
<td>17 %</td>
<td>17 %</td>
<td>19 %</td>
</tr>
</tbody>
</table>

Sources: Eurostat (National accounts, inflations, Euribor, Purchasing Power Parity (PPP), Gross Fixed Capital Formation (GFCF), population, Labour Force Survey -incl. Secondary job), Insee (Capital Stock Accounts)
Data are averaged from 1995 to 2007 to exclude the crisis. Depending on availability, samples may start after 1995.
EA (12) excl. FR stands for a 12-members Euro area excluding France and FR for France.
* in billion € in current prices
** annualised
*** aged from 15 to 64 in millions
† share of imports from EU partners in private consumption

Table 1: Actual data for France and the Euro Area and the corresponding endogenous values at steady state with our calibration
<table>
<thead>
<tr>
<th>Parameter Type</th>
<th>Parameter</th>
<th>France</th>
<th>Eurozone</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Structural parameters</strong></td>
<td></td>
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<td>Union-wide</td>
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<td>Technology parameter</td>
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<td>$\delta$</td>
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<td>Consensus</td>
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<tr>
<td>Capital rigidity</td>
<td>$S$</td>
<td>6.17</td>
<td>Smets and Wouters (2005)</td>
</tr>
<tr>
<td>Population size</td>
<td>$N^*$</td>
<td>135 922 100</td>
<td>ANA</td>
</tr>
<tr>
<td>TFP growth rate</td>
<td>$g$</td>
<td>0.003</td>
<td>ANA, Coenen et al. (2012)</td>
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<td>Monetary policy</td>
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<td></td>
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<tr>
<td>Inflation</td>
<td>$\Pi^*$</td>
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<td>Consensus, ECB</td>
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<tr>
<td>Smoothing parameter</td>
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<td>Barthélemy, Marx, and Poissonnier (2009)</td>
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<td>Weight on inflation</td>
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<td>Barthélemy et al. (2009)</td>
</tr>
<tr>
<td>Weight on output gap</td>
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<td>0.16</td>
<td>Barthélemy et al. (2009)</td>
</tr>
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<td>National specific</td>
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<tr>
<td>Population share</td>
<td>$n^i$</td>
<td>0.19</td>
<td>0.81</td>
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<tr>
<td>Trade openness</td>
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<td>Substitutability between goods</td>
<td>$\theta^i$</td>
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<td>Substitutability between workers</td>
<td>$\theta^w_i$</td>
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<td>4</td>
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<td>TFP scale factor</td>
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<td>Weight on labour disutility</td>
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<td>1.0003</td>
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<td>Government discount factor</td>
<td>$\beta^G$</td>
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<td>Inverse risk aversion</td>
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<td>1.13</td>
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<td>Inverse Frisch elasticity</td>
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<td>2</td>
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<td>Weight on public consumption utility</td>
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<td>Consumption habits</td>
<td>$h^c_i$</td>
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<td>0.61</td>
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<tr>
<td>Labour habits</td>
<td>$h^l_i$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Public consumption habits</td>
<td>$h^g_i$</td>
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<td>0.56</td>
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<td>Share of non-Ricardian agents</td>
<td>$\mu^i$</td>
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<td>0.4</td>
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<td>Price rigidity</td>
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<td>0.72</td>
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<tr>
<td>Wage rigidity</td>
<td>$\xi^w_i$</td>
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<td>0.72</td>
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<td>Price indexation</td>
<td>$\gamma^p_i$</td>
<td>0.13</td>
<td>0.13</td>
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<tr>
<td>Wage indexation</td>
<td>$\gamma^w_i$</td>
<td>0.36</td>
<td>0.36</td>
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<tr>
<td>Households financial premium</td>
<td>$\psi_c$</td>
<td>0.0005</td>
<td>0.0005</td>
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<tr>
<td>Government financial premium</td>
<td>$\psi_g$</td>
<td>0.0005</td>
<td>0.0005</td>
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<tr>
<td>Fiscal policy</td>
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<tr>
<td>Consumption tax rate</td>
<td>$\nu^c$</td>
<td>20.3%</td>
<td>19.5%</td>
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<tr>
<td>Labour tax rate</td>
<td>$\nu^w$</td>
<td>39.1%</td>
<td>37.7%</td>
</tr>
<tr>
<td>Capital tax rate</td>
<td>$\phi^k$</td>
<td>21.0%</td>
<td>17.0%</td>
</tr>
<tr>
<td>Transfers to GDP ratio</td>
<td>$\Phi$</td>
<td>19.4%</td>
<td>17.4%</td>
</tr>
</tbody>
</table>

ANA stands for Annual National Accounting data. For France, data are from the Insee, whereas international comparison within the Eurozone is conducted based on Eurostat data. Consensus indicates a value close to a large number of standard DSGE models.

Table 2: Structural parameters
In particular, the specification of households’ utility is a crucial determinant of the behaviour of the model. As highlighted in Everaert and Schule (2006), the estimation and identification of these two parameters, and in particular the (Frisch) elasticity of labour supply, is very sensitive to the methodology (micro or macro) and the sample considered.

Trabandt and Uhlig (2011) calibrate their model to an inverse Frisch elasticity of $\sigma_l = 1$ for France or the EU, in line with Kimball and Shapiro (2008) for the US. They also consider an alternative based on Cooley and Prescott (1995) with $\sigma_l = 0.33$ for the US. These values are in line with the business cycle literature and close to values estimated by Bayesian methods, as for instance in the different versions of Smets and Wouters’ model with $\sigma_l = 2.4$ (Smets and Wouters, 2003), $\sigma_l = 2.0$ (Smets and Wouters, 2005) both for the EU and $\sigma_l = 1.9$ (Smets and Wouters, 2007) for the US. However, micro and macro evidence is not easily reconciled and lead to very different values of the Frisch elasticity. Bayoumi et al. (2004) mention that micro studies give a range for $\sigma_l$ from 3 to as large as 20. In alternative scenarios for the GEM model, Bayoumi et al. (2004); Everaert and Schule (2006) set $\sigma_l = 6$ or 7 for Europe or France specifically. We calibrate MELEZE in line with Smets and Wouters (2005) in order to work with a medium range value of the Frisch elasticity, that is $\sigma_l = 2.0$.

For the inverse of the intertemporal consumption elasticity $\sigma_c$, the debate is less fierce and values range from 0.5 in Bayoumi et al. (2004) for EU countries to 2 as in Trabandt and Uhlig (2011) (EU and France). The different versions of Smets and Wouters give $\sigma_c = 1.3$ in Smets and Wouters (2003), $\sigma_c = 1.13$ in Smets and Wouters (2005) both for the EU and $\sigma_c = 1.4$ in Smets and Wouters (2007) for the US. We choose to match Smets and Wouters (2005) calibration with $\sigma_c = 1.13$.

Another weakly identified parameter, often estimated using Bayesian estimation methods or simply calibrated with “expert” insights, is the share of non-Ricardian households $\mu$. In GEM, this share is estimated to be 35% for France and 45% in the Euro area, whereas it stands to 40% for both in QUEST III. However, micro-studies highlight that these estimated shares might be over-evaluated as only a few agents are strictly banned from financial markets. Indeed, a large number of agents, designated as wealthy hand-to-mouth, do possess a large illiquid wealth, such as housing, so that their short-term consumption is highly correlated to their current income. However, in the long-term, this conclusion might differ as assets can be traded. Kaplan et al. (2014) compute values for the share of wealthy hand-to-mouth agents around 20% for France. Close to Kaplan et al. (2014), Martin and Philippon (2014) focus on the fraction of households with liquid assets representing less than 2 months of total gross income and calibrate their model to a 46.6% share of non-Ricardian agents in France. We choose to calibrate our model to estimated values in QUEST III.
Last, the calibration of the weight of public consumption in the utility is based on McGrattan et al. (1997), Bouakez and Rebei (2007) and Coenen et al. (2008). Estimating a CES public-private consumption aggregation, they identify a share of private consumption of 0.8 \((1 - \eta^i)\). In order to simplify the model, we resort to a Cobb-Douglas aggregation and therefore assume an elasticity of substitution of one.

However, this literature review is not enough to obtain a realistic calibration so that we proceed to an inverse inference. Indeed, in order to properly match the targeted observed levels or ratios in our model, a specific set of parameters needs to be optimized upon and cannot be set freely. For these parameters, we verify ex-post that they remain in the range identified in the literature. Table 1 compares for main structural levels and ratios their levels observed in the data with those generated by the model with our calibration. Targets based on observables are computed as averages over the period 1995-2007 to purposely exclude the crisis period.

First, the weights of leisure in the utility \(\kappa^i\) and the level of TFP in each country \(\zeta^i\) allow to replicate value added levels in each country \((\bar{Y^i})\), the terms of trade \((\bar{T})\) and hours worked \((\bar{L^i})\).

Second, the Cobb-Douglas parameter \(\alpha\) and the market power of firms \(\theta^i\) imply the distribution of output between the remuneration of capital and dividends (Gross Operating Surplus) and wages. There is however an imprecision in the data on GOS which includes mixed income, i.e. income of independent workers. Considering this and the wide range of mark-up rates in the literature we have not introduced differences across countries along this line.

Inflation is calibrated through the central banker’s target. The nominal interest rate is endogenous and clears the demand and supply of financial assets union-wide.

Final demands to GDP ratio are well replicated by the model. These ratios are endogenous: public consumption depends on the fiscal policy of both governments and balances their intertemporal budget constraint; investment depends on other parameters through equation (STEADY.5) and is actually little sensitive to the depreciation rate. Consumption clears the market once the trade balance is known.

The trade balance is related to the net financial asset position through aggregate national budget constraints. It can not be sizeable without much greater imbalances in terms of asset holdings. However, prices can differ across countries because of households preferences for domestic goods. These preferences are set such that we replicate the trade between France and its partners in the Euro Area and prices appear higher in France than for its partners in accordance with purchasing power parities data.
Discount factors $\beta^i$ and $\beta^{ig}$ are set to match observed debt-asset to GDP ratios in both countries (see equations (STEADY.1)). First note that it is not necessary for these factors to be lower than one, this condition applies to their transformation $\tilde{\beta}^i (1 + g)$ (Kocherlakota, 1990) and is verified by our calibration. Also, in a closed monetary union, it is not possible to replicate these ratios exactly as the Euro area is indebted vis-à-vis the rest of the world. We choose to impute the discrepancy to non French Euro Area residents who thus hold more assets in the model than in the data. In the end, the net financial asset positions of both countries mirror each other in the model and are rather small.

Taxes are set to the implicit tax rates on consumption and labour computed by Eurostat. Tax revenues from consumption are higher than in the data, but our models tax base does not discriminate between consumption and investment. Tax revenues from labour are lower than in the data, but our model does not treat the case of independent workers which may explain part of the discrepancy. As for capital, we only tax capital incomes in the model (return on capital, dividends and financial dividends) as opposed to capital stock. The tax rates on these three bases are supposed identical and set to approach capital tax income without prejudice on the investment to GDP ratio. These tax rates imply a tax revenue to GDP ratio close but lower than in the data, however, missing items in our simplified budget for the general government may account for this discrepancy.

4 Model dynamics

We analyse the short term properties of our model through the computation of IRFs for shocks occurring in France. We first plot the IRFs to standard shocks (productivity, preference, monetary policy), then to transitory fiscal shocks (VAT and government spending).

A detailed analysis of fiscal shocks in MELEZE including permanent ones, is given in Campagne and Poissonnier (2016a).

4.1 IRFs to standard shocks

Productivity shock  In Figure 2, productivity in France is increased by one percent with autocorrelation set to 0.9. This productivity shock implies an increase in output with marginal positive spillovers to the rest of the union. Prices drop in France but not in the rest of the Euro area, so that the central banker does not react much to this idiosyncratic shock and the real interest rate in France increases. Relative to financial savings, capital returns also benefit from the higher productivity. Hence investment increases to the detriment of financial savings. Private assets drop and symmetrically public debt as well, in France but not in the rest of the Euro Area. The drop in domestic prices is favourable to the terms of trade. However, the trade balance marginally depreciates as domestic demand is spurred while
Figure 2: IRFs to a one percent productivity shock (autocorrelated)

y-axis in p.p. deviation from steady state
demand addressed from the rest of the Euro Area is relatively unchanged. This is in part due to the
lower trade openness of the Eurozone with respect to France compared to the openness of France with
respect of the rest of the Eurozone.

In comparison with a closed economy model (Smets and Wouters, 2003, 2005), labour only temporary
decreases (the first year) as it becomes more productive. Simulations with a large weight of country one
in the monetary union show that differences are mainly due to the reaction of the central banker who
only slightly lowers its rate in comparison with a union wide productivity shock.

Preference shock In Figure 3, the discount factor in France is increased by one percent (i.e. more
patient households or a negative demand shock) with autocorrelation set to 0.9. As a consequence,
Ricardian households postpone consumption and increase their financial and physical savings. Prices
in France drop to limit the fall in final demand, so much so that the rest of the Euro Area follows
(though with delay and to a lesser extend), and the central banker sets an accommodative monetary
policy. The increase in investment more than compensates for the drop in consumption starting from
the second year after the shock. The drop in domestic demand relatively to the rest of the Euro Area is
beneficial to the trade balance, an effect magnified by the increase in the terms of trade.

Monetary policy shock In Figure 4, the monetary policy rate is increased by 100 basis points. This
large shock to monetary policy symmetrically impacts both France and the rest of the Euro Area. The
only difference stems from the financial asset holdings in both regions (and their consequences on the
trade balance and the terms of trade). Such a large tightening of monetary policy markedly depreciates
output union-wide. Prices plummet and gradually decrease both in France and in the rest of the Euro
Area, so much so that the endogenous behaviour of the central banker leads to a slight decrease in
its interest rate, partially offsetting its initial shock. Output reacts strongly negatively as the forward-
looking government cuts spendings reacting to the higher financing cost. A less adverse reaction is
observed when implementing a budget rule instead, as can be seen on Figure 9. These results are in line
with textbook simulations (Galí, 2008, Chapter 6).

Public spending Following a 1 percentage point autocorrelated increase in public spending (Figure 5),
output increases in France. With our Euler-type modelling of government consumption, this increase in
public spending is financed through public debt, mirrored by private assets. Prices marginally adjust
upwards to this demand shock and the central banker’s reaction to this idiosyncratic shock is minimal.
The demand shock is, by assumption, only addressed to domestic production and partially crowds out
investment. As a consequence, the demand addressed to the rest of the Euro Area increases but due to
price developments, the competitiveness of France deteriorates and the trade balance temporarily and
y-axis in p.p. deviation from steady state

Figure 3: IRFs to a one percent preference shock (autocorrelated)
Figure 4: IRFs to a 100 basis points monetary policy shock
Figure 5: IRFs to a one percent government spending shock (autocorrelated)
marginally declines.

The main difference with Smets and Wouters (2003) is the absence of crowding out on private consumption: private consumption slightly increases in reaction to the shock. This is linked to the non separability of government spendings and private consumption in the utility function and the introduction of non Ricardian households (see section 5.1 for more detailed explanation on the mechanisms at play).

5 Discussion of the model specification

We have introduced in the model several features which require a closer look: first, non Ricardian households and Edgeworth complementarity-substitutability are two competing mechanisms to generate stronger reaction of private expenditures to public spending shocks, second, the government maximizing households utility function departs from usual budget rules.

5.1 Non Ricardian households, Edgeworth complementarity/substitutability of private and public spending and their effect on consumption

Introducing non Ricardian households à la Campbell and Mankiw (1989) is advocated by Mankiw (2000) to analyse fiscal policy. When it comes to fiscal multipliers these agents can generate positive reaction of private consumption to public spending shocks in neo-Keynesian models (Galí, Vallés, and Lopez-Salido, 2007) as their marginal propensity to consume is 1. This stylised fact is not replicated in RBC models or standard neo-Keynesian models without non Ricardian households.

However, to replicate sizeable reactions, the share of non Ricardians may have to be set at a large value, inconsistent with microeconomic empirical facts. Fève and Sahuc (2013) consider Edgeworth complementarity in the utility function between private and public consumption as an other mechanism to generate a positive reaction of consumption to public spending. Their estimations of a closed economy model for the Euro Area favour this mechanism to non Ricardian households whose share is therefore estimated to be small (7%). With our general specification of utility, the two mechanism can be compared.

Mechanisms  Without either non Ricardian households or Edgeworth complementarity-substitutability, government spending shocks tend to crowd out private consumption through its inflationary effect. Non Ricardian households contrary to Ricardian ones can not save the increased income implied by higher government expenditure: they can not postpone consumption. Hence if the fraction of non Ricardians is sufficiently large a positive reaction of private consumption to a government spending shock. With Edgeworth complementarity-substitutability only, government spending directly affects households marginal
utility and weights on all their arbitrages. However, assuming substitutability or complementarity will imply opposite reactions.

To exemplify the implications of substitutability or complementarity in the Edgeworth mechanism, we change the intertemporal elasticity of substitution of consumption.

With our baseline calibration ($\sigma > 1$), the marginal utility of consumption decreases with public spendings, hence private and public consumption are Edgeworth substitute. Following a positive shock to government spending, ceteris paribus households will have to compensate by lowering private consumption to readjust the marginal utility of consumption in their different arbitrages. With an alternative calibration ($\sigma_c = 0.6 < 1$), private and public consumption are Edgeworth complements: following a positive shock to government spending, the utility functional form implies that the marginal utility of consumption increases, so that households will increase their consumption to decrease this marginal utility in their different arbitrages.

**Simulations** Figure 6 exemplifies these mechanisms. We simulate a one point shock to public spending, with autocorrelation 0.9.

In our baseline scenario, we assume $\sigma_c > 1$, resulting also in an Edgeworth substitutability. As a result, the two mechanisms under scrutiny work in opposite directions (Figure 6a). With non Ricardian households only, consumption reacts positively, whereas with Edgeworth substitutability only, consumption reacts negatively upon shock.

With our calibration, the effect of non Ricardian households dominates Edgeworth substitutability. This result crucially depends on the parameters of the utility function and the share of non Ricardians. With neither mechanism, consumption reacts negatively upon shock (although less than with Edgeworth substitutability), with both it reacts similarly to the case with non Ricardians only.

In the case of $\sigma_c < 1$ (Figure 6b), assuming none of the two mechanisms generates the usual crowding out effect and a negative response of consumption upon shock. Edgeworth complementarity and non Ricardian households both generate a (small) positive reaction of private consumption upon shock. With both mechanisms (baseline scenario), the two effects are combined, though dominated by the non Ricardian agents mechanism, and private consumption increases by one tenth of public spending deviation.

5.2 Government spending in the utility function compared to budget rules

Corsetti et al. (2010) investigate the effect of fiscal stimulus in a two country model (not in a monetary union). Their investigation of cross border spillovers is based on a pair of budget and transfer rules
endogenizing government spending and tax instruments in order to ensure long-term solvency of the government. They show that the impulse responses to government spending shocks and the associated fiscal multipliers depend markedly on the combination of instruments used to ensure this long term solvency (lump-sum taxes or spending) and on the sensitivity of these instruments to government spending and output gap respectively.

As we introduce government spending in the utility function, we explained that the government can be modelled as an optimizing agent with an explicit objective, and we now compare this behaviour with Corsetti et al.’s spending rule. As they do, we consider a pro or contra cyclical rule as well as an acyclic one (see 2.4). Figures 7, 8 and 9 present the Impulse Response Functions for productivity, preference and monetary policy shocks.

Two main analytical differences between the budget rule and the optimizing government are the reaction to interest rates and the forward looking set-up of the optimizing government. Second round effects in the Euler equation through labour and private consumption are of secondary importance. Because the optimizing government is forward looking, government spending is directly adjusted upon shock (see for instance Figure 7). The reaction to interest rates (the cost of public debt) is also sizeable (Figure 9).

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\[^{21}\text{Corsetti et al. (2010) do not introduce a spread on public debt as in MELEZE.}\]

\[^{22}\text{Figures 10 to 12 in appendix provide complementary IRFs.}\]
Despite these differences, the IRFs of output and the real interest rate are quite similar between both 
types of modelling paradigms. The main differences are indeed with public consumption and public 
debt. The IRFs related to the three different government spending rules (pro-cyclical, contra-cyclical and 
acyclic) exhibit little differences with one another. In comparison with the optimizing government, all 
three exhibit sizeable oscillations around the steady state, showing weak convergence mechanisms.23 
The optimizing government shows monotonic IRFs.

All in all, these simulations show that although budget rules are a simple and flexible way to de-
scribe the government’s behaviour, one need to be careful when including such rules into a general 
equilibrium model. However, differences in IRFs are confined to the government block and do not 
largely influence the behaviour of other real variables across types of modelling but also across cyclical-
ity of the budget rules.

Figures 13 and 14 show similar results for a government consumption shock.

---

23 Other simulations show that this behaviour is amplified if the reaction to the deviation of public debt to its target is stronger 
and is linked to the autocorrelation of the budget rule: when it is smaller, fluctuations fade out.
y-axis in p.p. deviation from steady state

Figure 8: IRFs to a one percent preference shock (autocorrelated)
y-axis in p.p. deviation from steady state

Figure 9: IRFs to a 100 basis points monetary policy shock
References


M. Kimball and M. Shapiro. Labor supply: Are the income and substitution effects both large or both small? *NBER Working Paper Series*, 14208(July), 2008.


A Steady state

We use \( \bar{\cdot} \) to denote the steady state operator. It can be defined only for detrended real variables and inflations or relative prices as there is exogenous growth and non zero inflation.

We denote \( c_y^i = \bar{C}^i / \bar{Y}^i \), \( g_y^i = \bar{G}^i / \bar{Y}^i \) and \( i_y^i = \bar{I}^i / \bar{Y}^i \), the shares of private consumption, public expenditure and investment in each country’s GDP, and \( \theta = \frac{Y_1}{Y_2} \), the relative size of production of country 1 and 2.

At steady state, we assume that inflation and monetary interest rate equal the central bank’s targets.

A.1 Goods Aggregation

The different consumptions are linked as follows:

\[
\bar{C}_1^i = (1 - \alpha^1)C_1^1 T^{\alpha^1} \quad \bar{C}_2^i = \alpha^1 C_1^1 T^{\alpha^1 - 1} \quad (A.1, A.2)
\]

\[
\bar{C}_2^i = \alpha^2 C_2^1 T^{1 - \alpha^2} \quad \bar{C}_2^i = (1 - \alpha^2) C_2^1 T^{\alpha^2 - 1} \quad (A.3, A.4)
\]

Identical equalities hold for investment. Aggregation across households reads:

\[
\bar{C}^i = \bar{C}^{NR,i} + \bar{C}^{R,i} \quad (A.5)
\]

A.2 Households

A.2.1 Ricardian Households Euler equation

From the Euler equation of Ricardian households and governments, we have:

\[
\bar{\beta}^i \bar{R} - \psi (\bar{d}^i) \bar{\Pi} = 1 \quad (STEADY.1)
\]

where we define \( \bar{\beta}^i = \beta^i (1 + g)^{(1 - c^i)((1 - \eta)(1 - h_2) + \eta(1 - h_1)) - 1} \) with \( g \) the exogenous growth rate of TFP.

A.2.2 Capital, Investment and Tobin’s Q

At steady state, the capital dynamics, the investment decision and Tobin’s Q are defined by:

\[
\bar{I}^i = (\delta + g) \bar{K}^i \quad (A.6)
\]

\[
1 = \bar{\beta}^i \left( (1 - \delta) + \frac{(1 - \rho^{K,i}) p^{K,i}}{1 + \rho^{c,i}} \right) \quad (A.7)
\]

\[
\bar{q} = 1 \quad (A.8)
\]
From the previous equation we can isolate the interest rate on capital:

$$\bar{r}^{K,j} = \left( \frac{1}{\beta^j} - 1 + \delta \right) \frac{1 + \bar{v}^{r,j}}{(1 - \bar{v}^{r,j})}$$ (STEADY.2)

### A.2.3 Households’ trade-off between consumption and leisure

For households’ trade-off between consumption and leisure not to be distorted by long term growth, the ratio of marginal utilities of labour and consumption must have the same trend as real wages. This is possible either with non separable utility or log utility of consumption (King et al., 2002).

This arbitrage is described by the Phillips curve on wages and yields:

$$\frac{\bar{W}^{R,j}L^{R,j}}{\bar{C}^{R,j}} = \frac{\theta^j_w (1 + \sigma^j)\sigma^j_c}{\theta^j_w - 1} \left( \frac{1 + \sigma^j}{1 - \sigma^j} \right) \frac{\kappa^j (1 - \sigma^j)}{1 - \kappa^j (1 - \sigma^j)} \left[ \frac{\bar{L}^{R,j}}{(1 - \mu^j)^{1/N}} \left( \frac{L^i}{\pi^R} \right) - b_i \right]^{1 + \sigma^j}$$ (A.9)

$$\frac{\bar{W}^{NR,j}L^{NR,j}}{\bar{C}^{NR,j}} = \frac{\theta^j_{wl} (1 + \sigma^j)\sigma^j_c}{\theta^j_{wl} - 1} \left( \frac{1 + \sigma^j}{1 - \sigma^j} \right) \frac{\kappa^j (1 - \sigma^j)}{1 - \kappa^j (1 - \sigma^j)} \left[ \frac{\bar{L}^{NR,j}}{(1 - \mu^j)^{1/N}} \left( \frac{L^i}{\pi^N} \right) - b_i \right]^{1 + \sigma^j}$$ (A.10)

which can be later on simplified using the share of both types of households in consumption and payroll, the share of consumption in output and the share of payroll in output.

### A.2.4 Ricardians’ and non Ricardians’ shares of consumption and payroll

We showed that consumption and payroll of Ricardians and non Ricardians add up to total consumption and payroll in each country at each date. We denote the shares of non Ricardians in consumption $s^j_{wc} = \frac{C^{NR,j}}{C^R}$, the shares in payroll $s^j_{wl} = \frac{W^{NR,j}}{W^R}$.

The Dixit-Stiglitz aggregation of labour in both countries yields:

$$\frac{\bar{W}^{R,j}L^{R,j}}{\bar{W}^j} = \left( 1 - s^j_{wl} \right) \frac{1}{1 - \mu^j} \frac{1}{\sigma^j_{wl} - 1}$$ (A.11, A.12)

$$\frac{\bar{W}^{NR,j}L^{NR,j}}{\bar{W}^j} = \mu^j \left( 1 - s^j_{wl} \right) \frac{1}{\sigma^j_{wl} - 1}$$ (A.13, A.14)
The budget constraint of non Ricardians relates the two shares in the following way:

\[
\text{RPC}^j cy^j s^j_r = \frac{1 - \alpha}{(1 + \varphi^e)^{(1 + \varphi^w)}} \left( (\theta^i - 1) \theta^i s^i_{w} + \frac{s^N \rho_{i,j}^N}{1 + \varphi^i} \right) \tag{STEADY.3}
\]

The budget constraint of Ricardians is collinear to this equation and does not help in calibrating the shares (see E).

At steady state, the consumption-leisure arbitrages combined also relate the two shares:

\[
\left( \frac{s^j_{w}}{1 - s^j_{w}} \right)^{(1+\varphi^w)\frac{1+\varphi^w}{\theta^w(1-\theta^w)}} = \left( \frac{\mu^i}{1 - \mu^i} \right) \left( 1 + \frac{1 - s^i_{w} \theta^i - 1}{s^i_{w} \theta^i - 1} \theta^i \frac{1 - s^i_{w} \theta^i - 1}{s^i_{w} \theta^i - 1} \theta^i \frac{1 - \alpha}{(1 + \varphi^e)(1 + \varphi^w)} \frac{1}{\alpha^w} \frac{1}{(1 + \varphi^e)(1 + \varphi^w)} \right) \tag{STEADY.4}
\]

Together, equations (STEADY.3) and (STEADY.4) define the shares of both types of households in consumption and payroll as functions of the other parameters in the model, first of which \(\mu^i\). This system must be solved numerically once other parameters are known. Note that if \(\mu^i = 0\), \(s^i_{w} = s^i_{w} = 0\) is indeed a solution to this system when no transfers are paid to the (empty) population of non Ricardians.

### A.3 Firms

From the production function, we have:

\[
Y^i = (\bar{L}^i)^{1-a} (R^i)^{a} \bar{X}^i \tag{A.15}
\]

with \(\bar{L}^i\) the steady state value of detrended TFP, a scale factor to be determined, and \(\bar{X}^i = 1\), see Appendix D. Note that due to our time to build assumption on capital, capital stock at time \(t - 1\) is detrended with TFP trend at time \(t\).

From the arbitrage between capital and labour input we already have:

\[
\frac{1 - a}{a} = \frac{RW^j (1 + \varphi^e)^{(1 + \varphi^w)}L^i}{p^j K^i} \tag{A.16}
\]

with \(RW^j = \frac{W^{i}}{CFT(1+\varphi^w)T^j}\) the purchasing power of net wages in country \(i\).
From the Phillips curve on prices, at steady state the mark-up, i.e. the ratio of production price over the marginal cost of production equals \( \frac{\theta_i^*}{\theta_i^*} \).

\[
\frac{\text{RPC}}{\text{RMC}} = \left( \frac{\text{RW}^i}{\text{Y}^i} (1 + \bar{\nu}_w^i) (1 + \bar{\nu}_c^i) \right)^{1-\alpha} \left( p^k_i \right)^{\alpha} = \alpha^{\alpha - 1} \theta_i^* \tag{A.17}
\]

\[
\frac{\theta_i^*}{\theta_i} - 1 \theta_i^* \tag{A.18}
\]

Also, because of constant returns to scale, we can check that the mean cost equals the marginal cost and we find that:

\[
\frac{\text{RPC}}{\text{RMC}} \frac{\text{RW}^i}{\text{Y}^i} (1 + \bar{\nu}_w^i) (1 + \bar{\nu}_c^i) \bar{L}^i = (1 - \alpha) \frac{\theta_i^* - 1}{\theta_i} \tag{A.19}
\]

\[
\frac{\text{RPC}}{\text{Y}^i} = \frac{\theta_i^* - 1}{\theta_i} \tag{A.20}
\]

Combining the capital share with the steady state equations for capital dynamic and investment decision (section A.2.2) gives:

\[
\alpha \frac{\theta_i^* - 1}{\theta_i} \theta_i^* = \left( \frac{1}{\bar{\beta}^i} - (1 - \delta) \right) \frac{1 + p^k_i}{1 - p^k_i} \frac{\text{RPC}^i}{\text{RMC}^i} \tag{STEADY.5}
\]

The firms’ dividends equation gives:

\[
\frac{D^i}{p^i Y^i Tr} = \bar{d}^i = 1 - \left( \frac{\text{RW}^i}{\text{Y}^i} (1 + \bar{\nu}_w^i) (1 + \bar{\nu}_c^i) \frac{\bar{L}^i}{\bar{Y}^i} + p^k_i \frac{\bar{K}^i}{\bar{Y}^i} \right) \text{RPC}^i \tag{A.21}
\]

\[
\bar{d}^i = 1 - \frac{1}{1 - \alpha} \frac{\text{RW}^i}{\text{Y}^i} (1 + \bar{\nu}_w^i) (1 + \bar{\nu}_c^i) \frac{\bar{L}^i}{\bar{Y}^i} \text{RPC}^i \tag{A.22}
\]

that rewrites with help of the Phillips curve on prices and arbitrage between K and L:

\[
\bar{d}^i = 1 + \frac{1 - \theta_i^*}{\theta_i} \theta_i^* = \frac{1}{\theta_i^*} \tag{A.23}
\]

Firms are able to fix prices higher than their marginal cost and keep a mark-up \( \frac{1}{\theta_i^*} \).
A.4 Fiscal Authorities

With our maximizing government, the public debt level is pinned down thanks to the debt elastic spread and from the Euler equation of governments, we obtain:

\[ \tilde{\beta}_g \bar{R} - \psi(\bar{p}a^1) - \psi'(\bar{p}a^i)\bar{p}a^i = 1 \]  

(STEADY.6)

where we define \( \tilde{\beta}_g = \beta_g(1+g)^{(1-\psi(\bar{p}a^1))((1-\eta)(1-\bar{h}^c)+\eta(1-h^c))^{-1}} \).

A.5 Financial Intermediation

Financial intermediaries’ profits write:

\[ \tilde{f}_d^1 = \frac{\theta f_a}{1 + \theta f_a} \frac{FY}{p^1 Y^1 Tr} \]

(A.24)

\[ \tilde{f}_d^2 = \frac{1}{1 + \theta f_a} \frac{FY}{p^2 Y^2 Tr} \]

(A.25)

In addition, the zero cash need condition implies at all time and a fortiori at steady state that the net financial position of one country neutralises the net financial position of the other:

\[ FA^1_t + FA^2_t + PA^1_t + PA^2_t = 0 \]  

(A.28)

\[ f_a^1 + pa^1 = -\frac{f_a^2 + \bar{p}a^2}{\bar{T}} \]  

(STEADY.7)
A.6 Monetary Authority, Prices and Inflations

Using the definition for both the relative price of consumption and the consumption price index, we have the following relationships:

\[ \overline{RPC}_1 = \bar{T}^{a_1} \quad \overline{RPC}_2 = \bar{T}^{-a_2} \quad \text{(STEADY.8)} \]

Inflation rates are also directly related:

\[ \Pi^{C,1} = \Pi^{1-a_1,1} \quad \Pi^{C,2} = \Pi^{2-a_2,2} \quad \text{(A.29, A.30)} \]

As a result, if \( \bar{T} \) exist, that is if the ratio of consumption and production prices or domestic and importation prices has a steady state value in both countries, a solution exists such that:

\[ \Pi^1 = \Pi^2 = \Pi^{C,1} = \Pi^{C,2} \quad \text{(A.31)} \]

We assume that such an inflation level is the central bank’s target. Finally, from the Taylor rule we have:

\[ R = R^*, \text{ since } \bar{Y} = 1 \text{ and } \Pi = \Pi^* \quad \text{(A.32)} \]

i.e. the central banker set the interest rate and inflation at its target level at the steady-state.

A.7 Market Clearing

At steady state, we have from the market clearing equation

\[ \bar{Y}^1 = \bar{C}_1^1 + \bar{C}_1^2 + \bar{G}_1^1 + \bar{I}_1^1 + \bar{I}_1^2 \quad \text{(A.33)} \]
\[ \bar{Y}^2 = \bar{C}_2^2 + \bar{C}_2^1 + \bar{G}_2^2 + \bar{I}_2^2 + \bar{I}_2^1 \quad \text{(A.34)} \]

These equations imply that the market clearing condition at steady state is:

\[ 1 = (cy^1 + iy^1)\bar{T}^{a_1} + gy^1 + \bar{I} \quad 1 = (cy^2 + iy^2)\bar{T}^{-a_2} + gy^2 - \frac{\theta}{\bar{T}}\bar{I} \quad \text{(STEADY.9)} \]
A.8 Budget constraints

From the budget constraint of Ricardian households, we obtain in real terms:

\[ fa_i' = \frac{FA_i'}{P_i'Y_i'T_i} = \left( R_{i,-1} - \psi(fa_{i,-1}') \right) \frac{fa_{i,-1}'}{\Pi_i' (1 + g)} + RPC_i' \left( 1 + \psi^{c,i}_I \right) \frac{RW_i' R_{i,j} L_{i,j}'}{Y_i'T_i} \]

\[ - RPC_i' \left( 1 + \psi^{c,i}_I \right) \frac{C_{i,j}'}{Y_i'T_i} + \left( 1 - \psi^{FD,i}_I \right) d_i' + \left( 1 - \psi^{FD,i}_I \right) d_i' + \Phi^{R,j}_I \]

\[ - RPC_i' \left( 1 + \psi^{c,i}_I \right) \frac{I_i'}{Y_i'T_i} + RPC_i' \left( 1 - \psi^{K,i}_I \right) r_i' \]

(\ref{A.35})

denoting \( \Phi^{R,j}_i = \frac{\Phi^{R,j}_i}{\Pi_i' Y_i}. \) At steady state:

\[ 0 = \left( R - \Pi_i' (1 + g) - \psi(fa_i') \right) \frac{fa_i'}{\Pi_i' (1 + g)} + \frac{1 + \psi^{c,i}_I}{Y_i'} \left[ RW_i' R_{i,j} L_{i,j}' - C_{i,j}' \right] RPC_i' \]

\[ + \left( 1 - \psi^{FD,i}_I \right) \frac{f_i'}{Y_i'} + \left( 1 - \psi^{FD,i}_I \right) \frac{d_i'}{Y_i'} + \Phi^{R,j}_I \]

\[ + \left( 1 - \psi^{K,i}_I \right) r_i' \frac{R_{i,j}'}{Y_i} RPC_i' - \left( 1 + \psi^{c,i}_I \right) \frac{I_i'}{Y_i} RPC_i' \]

(\ref{A.36})

which gives the following relationship:

\[ fa_i = \frac{\bar{\beta}^i (1 + g)}{1 - \beta (1 + g)} \left\{ \left( 1 + \psi^{c,i}_I \right) RPC_i' \left( (1 - s_i^c I') c y_i' + i y_i' \right) - \frac{1 - \psi^{D,i}_I}{\theta_i'} - \left( 1 - \psi^{FD,i}_I \right) \frac{d_i'}{Y_i'} - \Phi^{R,j}_I \right\} - \left( 1 - \alpha \right) \frac{\theta_i^I - 1}{\theta_i^I} \frac{1 - s_i^w I}{\theta_i^I} \]

\[ = \left( 1 - \alpha \right) \frac{\theta_i^I - 1}{\theta_i^I} \frac{1 - s_i^w I}{\theta_i^I} \]

(STEADY.10)

As for non Ricardian households, we have:

\[ \frac{\bar{C}^{NR,j}_{i}}{Y_i'} = \frac{RW_i^{NR,j} L_{i,j}^{NR,j}}{Y_i'} + \frac{\Phi^{NR,j}_i}{1 + \psi^{c,i}_I} \frac{1}{RPC_i'} \]

(\ref{A.37})

\[ RPC_i' \frac{c y_i' s_i^c}{\left( 1 + \psi^{c,i}_I \right) \left( \theta_i^I - 1 \right)} = \frac{1 - \alpha}{\left( 1 + \psi^{c,i}_I \right) \left( \theta_i^I - 1 \right)} \frac{\theta_i^I - 1}{\theta_i^I} s_i^w I + \frac{\Phi^{NR,j}_i}{1 + \psi^{c,i}_I} \]

(STEADY.11)
Identically the budget constraint of governments gives in real terms:

\[ pa^i = \frac{PA^i}{\bar{p}^i} = \left( R_{i-1} - \psi^{\theta}(pa_{i-1}) \right) \frac{pa^i_{i-1}}{\bar{p}^i(1 + g)} + \frac{\nu^d_j f d^i_t}{\bar{p}^i} + \frac{\nu^d_j f d^i_t}{\bar{p}^i} - \frac{C^i_{i-1}}{\bar{p}^i} + \phi^i \]

(A.38)

and defining \( pa^i = \frac{pa^i}{\bar{p}^i} \), we obtain at steady state:

\[ gy^i + \psi^i = \left( \frac{1}{\bar{g}^i(1 + g)} - 1 + \frac{\nu^d_j \psi^{\theta}(pa^i)}{\bar{p}^i(1 + g)} \right) \bar{p}^i + \frac{\nu^d_j \bar{R}^i C^i(2)}{\bar{p}^i} + \frac{\nu^d_j f d^i_t}{\bar{p}^i} + \left( \frac{\nu^d_j}{\bar{p}^i} \right) \left( -\psi^{\theta} + (1 - \alpha) + \nu^d_j \alpha \right) \]

(STEADY.12)

### A.9 Trade balance

Let \( TB_i = p^i_1 C^i_{1,i} + p^i_2 C^i_{2,i} - p^i_1 l^i_{1,i} + p^i_2 l^i_{2,i} \) denote the trade balance of country 1. At steady state

\[ \frac{TB_i}{p_1 Y_I} = \bar{t}b = \frac{C^2_1 + l^i_{1,i}}{Y_I} - \frac{C^1_1 + l^i_{1,i}}{Y_I} \]

(A.39)

and the openness of the two countries verifies:

\[ \bar{t}b = \alpha^2 \frac{cy^2 + iy^2}{\theta} - \alpha^2 - \alpha \left( cy^2 + iy^2 \right) \bar{p}^i \]

(STEADY.13)

Combining the three budget constraints of a country yields, after simplification of transfers and taxes, combination of Ricardian and non Ricardian households:

\[ FA^i + PA^i = R_{i-1}(FA^i_{i-1} + PA^i_{i-1}) + TB_i + FD^i_t - \psi(fa^i_{i-1})FA^i_{i-1} - \psi(pa^i_{i-1})PA^i_{i-1} \]

(A.40)

That is in real terms:

\[ fa^i + pa^i = R_{i-1}(fa^i_{i-1} + pa^i_{i-1}) \frac{1}{\Pi^i_t(1 + g)} + \bar{t}b + \frac{fd^i_t}{\bar{p}^i} \]

- \psi(fa^i_{i-1})fa^i_{i-1} \frac{1}{\Pi^i_t(1 + g)} - \psi(pa^i_{i-1})pa^i_{i-1} \frac{1}{\Pi^i_t(1 + g)} \]

(A.41)

In words, the current account (changes in the net financial position of the country as a whole) depends on its trade balance and net transfers with the rest of the union (interests paid on its position in the pre-
vious period spread included net of dividends received from the international financial intermediary). We assume that at steady state the spread paid to the financial intermediary is offset by the dividends received implying the following for \( \theta^{fa} \):

\[
\theta^{fa} = \frac{\theta \psi(fa^1)fa^1 + \psi^g(pa^1)pa^1}{\bar{T} \psi(fa^2)fa^2 + \psi^g(pa^2)pa^2}
\]  

(A.42)

It follows at steady state that:

\[
\bar{tb} = \left(1 - \frac{\bar{R}}{\bar{\Pi}(1 + g)}\right) (fa^1 + pa^1)
\]  

(STEADY.14)

Given the choice of redistribution, financial dividends simplify at steady state into:

\[
\bar{fd}^1 = \frac{1}{\bar{T}(1 + g)} \left(\psi(fa^1)fa^1 + \psi^g(pa^1)pa^1\right); \bar{fd}^2 = \frac{1}{\bar{T}^2(1 + g)} \left(\psi(fa^2)fa^2 + \psi^g(pa^2)pa^2\right)
\]  

(STEADY.15)

### A.10 Endogenous variables in level

In the present model, growth is exogenous. In the long run, all real variables grow at the rate of TFP and prices grow at the steady state inflation rate. Taking into account all steady state equations shows that it is possible (and necessary) to define all endogenous variables not only in ratio to GDP but also in level.

Conveniently, for policy analysis purposes, this allows to assess how this steady state in level depends on structural, technological, preference and fiscal parameters and by derivation how changes in these parameters would influence the level of our economies.
Starting from the Ricardian households consumption-leisure trade-off (equation (A.10)), we get:

\[
\frac{\bar{R}^k_j}{\bar{C}^k_j} \left( \frac{L^k_j}{1 - \sigma^k_j} \right) = \frac{\theta^k_{\ell_0} - 1}{\theta^k_{\ell_0} - 1} \kappa^k (1 - \sigma^k_j) \left[ \frac{L^k_{R_j}}{(1 - \mu)^{N\bar{R}}} \left( \frac{\bar{L}_i}{\bar{R}^{N\bar{R} + 1}} \right) \right]^{1 + \sigma^k_j} \]  

(A.43)

\[
\frac{1 - s^i_{w,j}}{1 - s^i_{c}} \left( \frac{R^i}{C^i} \right) = \frac{\theta^i_{\ell_0} - 1}{\theta^i_{\ell_0} - 1} \kappa^i (1 - \sigma^i_j) \left[ \frac{L^i_{R_j}}{(1 - \mu)^{N\bar{R}}} \left( \frac{\bar{L}_i}{\bar{R}^{N\bar{R} + 1}} \right) \right]^{1 + \sigma^i_j} \]  

(A.44)

that is

\[
L^i = \left( \frac{1 - \mu^i}{\bar{R}^{N\bar{R} + 1}} \right)^{1 + \sigma^i_j} \left( \frac{1}{(1 - \eta^i) L^i} \right)^{1 + \sigma^i_j - 1} \left( \frac{\rho^i_{\ell_0}}{\rho^i_{\ell_0} - 1} \right)^{1 + \sigma^i_j} \left( \frac{B^R_i}{B^R_j} \right)^{1 + \sigma^i_j - 1} \]  

(STEADY.16)

with \(B^R_i = \frac{1 - s^i_{w,j}}{1 - s^i_{c}} \theta^i_{\ell_0} - 1 \frac{1}{\rho^i_{\ell_0} - 1} \) and \(k^i = \kappa^i (1 - \sigma^i_j) (n^i N)^{-1 - h^i} \) \( (1 + \sigma^i_j) \).

To determine the level of output we write from the Cobb-Douglas function, simplifying with (STEADY.5):

\[
Y^i = (\bar{R}^k)^{\alpha} (\bar{C}^k) (\bar{L}^i)^{1 - \alpha} = \bar{Z}^i \left( \frac{\bar{L}^i}{\bar{B}^i} \right)^{\frac{1}{\beta + \gamma}} \left( \frac{\bar{L}^i}{\bar{B}^i} \right)^{\frac{1}{\beta + \gamma}} \]  

(A.45)

which we simplify in:

\[
Y^i = AZ^i \]  

(STEADY.17)

From equations (STEADY.17) and (STEADY.16) we also have the following purchasing power of wages:

\[
\bar{R}W^i = A \frac{(1 - \alpha)}{(1 + \rho^k_j)} \frac{1}{\rho^\ell_0} \]  

(STEADY.18)

61
Finally, non Ricardians and Ricardians shares in the population and from equations (STEADY.3) and (STEADY.4) in consumption and payroll allow to compute the steady state level of consumption, labour supply and wage of both types of households. Similarly, from equation (STEADY.17) the steady state levels of consumption, government spending, investment, capital and the trade balance are directly found by multiplying by $cy^i$, $gy^i$, $iy^i$, $\delta^i$ and $\bar{tb}$ respectively. Private assets, public assets, transfers and financial dividends in real terms can be found by multiplying $\bar{Y}^i$ with $fa^i$, $pa^i$, $\phi^i$, and $fd^i$. Dividends from the real sector are one $\theta$th of output. Openness degrees ($a^j$) allow to compute the share of investment and consumption imported from the rest of the union.

B Linearisation

This appendix presents the full linearisation of the model. Equations labelled LIN are equations included in the corresponding Dynare code (available upon request).

B.1 Goods Aggregation

Linearising the relationships in Section 2.1 gives:

$$
\hat{C}^1_{1,t} = a^1 \hat{T}_t + \hat{C}^1_t \\
\hat{C}^2_{1,t} = (a^2 - 1) \hat{T}_t + \hat{C}^2_t \\
\hat{C}^1_{2,t} = (1 - a^2) \hat{T}_t + \hat{C}^1_t \\
\hat{C}^2_{2,t} = -a^2 \hat{T}_t + \hat{C}^2_t
$$

(LIN.1, LIN.2)

(LIN.3, LIN.4)

Identical equalities hold for investment.

$$
\hat{I}^1_{1,t} = a^1 \hat{T}_t + \hat{I}^1_t \\
\hat{I}^1_{2,t} = (a^1 - 1) \hat{T}_t + \hat{I}^1_t \\
\hat{I}^2_{1,t} = (1 - a^1) \hat{T}_t + \hat{I}^2_t \\
\hat{I}^2_{2,t} = -a^1 \hat{T}_t + \hat{I}^2_t
$$

(LIN.5, LIN.6)

(LIN.7, LIN.8)

---

$24 \hat{X}$ is variable $X$’s log-deviation from its steady state value $\bar{X}$. 

---
B.2 Households

B.2.1 Ricardian Households Euler Equations

The Euler equation for Ricardian households is given by equation (2.25). Taking the log-linearisation around the steady-state and simplifying the relationship yields:

\[ 0 = \left(1 - (1 - \sigma_t^j)(1 - \eta)\right) \left[ \tilde{C}^{R,j}_{t+1} - \tilde{C}^R_{t+1} \right] + h^j_1(1 - \sigma_t^j)(1 - \eta) \left[ \tilde{C}_t - \tilde{C}_{t-1} \right] \\
- (1 - \sigma_t^j) \left[ \tilde{C}^{R,j}_{t+1} - \tilde{C}^j_{t+1} \right] + h^j_s(1 - \sigma_t^j) \eta \left[ \tilde{C}_t - \tilde{C}_{t-1} \right] \\
+ (1 + \sigma_t^j) \alpha_t^j B^{R,j} \left[ (\hat{L}^{R,j}_{t+1} - \hat{L}^R_{t+1}) - \hat{h}^j_1(\hat{L}_t - \hat{L}_{t-1}) \right] + \hat{I}^{j}_{t+1} + \frac{\varphi_{c,j}^j}{1 + \varphi_{c,j}^j} (\tilde{c}^j_{t+1} - \tilde{c}^j_t) \quad \text{(LIN.9)} \]

\[ \frac{1}{R - \psi(fa^i)} \left( R\hat{R}_t - \psi(fa^i)fa^i\hat{f}_a^i \right) \]

B.2.2 Tobin’s Q and investment decision

Linearising around the steady-state and simplifying the relationships yield:

\[ 0 = \beta_t^j + \beta_t^j \hat{I}^j_{t+1} - S''(1 + g)(1 + g)^2 \left( \hat{I}_t^j - \hat{I}_{t+1} - \beta_t^j(1 + g)(\hat{I}_{t+1} - \hat{I}_t) \right) \quad \text{(LIN.10)} \]

\[ \hat{q}_t^j = \left(1 - \sigma_t^j\right)(1 - \eta) \left[ \tilde{C}^{R,j}_{t+1} - \tilde{C}^j_{t+1} \right] - h^j_1(1 - \sigma_t^j)(1 - \eta) \left[ \tilde{C}_t - \tilde{C}_{t-1} \right] \\
+ (1 - \sigma_t^j) \eta \left[ \tilde{C}^{R,j}_{t+1} - \tilde{C}^j_{t+1} \right] + h^j_1(1 - \sigma_t^j) \eta \left[ \tilde{C}_t - \tilde{C}_{t-1} \right] \\
- (1 + \sigma_t^j) \alpha_t^j B^{R,j} \left[ (\hat{L}^{R,j}_{t+1} - \hat{L}^R_{t+1}) - \hat{h}^j_1(\hat{L}_t - \hat{L}_{t-1}) \right] \quad \text{(LIN.11)} \]

\[ + \beta_t^j(1 - \delta) \hat{q}_{t+1} + \beta_t^j \hat{q}^{k,j}_{t+1} \frac{(1 - \varphi_{c,j}^j)}{1 + \varphi_{c,j}^j} \left[ \hat{k}_{t+1} - \hat{q}_{t+1} \right] \]

B.2.3 Capital dynamics

The dynamics of capital is given by:

\[ (1 + g)\hat{K}^j_t = (1 - \delta)\hat{K}^j_{t-1} + (\delta + g)(\hat{I}_t + \hat{e}_t^j) \quad \text{(LIN.12)} \]

B.2.4 Ricardians’ and non Ricardians’ aggregation
\[
\hat{C}_i^j = s_i^j \hat{C}_{i}^{NR} + (1 - s_i^j) \hat{C}_{i}^{R} \tag{LIN.13}
\]
\[
\hat{C}_i^j = s_i^j \hat{C}_{i}^{NR,j} + (1 - s_i^j) \hat{C}_{i}^{R,j} \tag{LIN.14}
\]
\[
\hat{R}W_i^j = s_i^j \hat{R}W_{i}^{NR,j} + (1 - s_i^j) \hat{R}W_{i}^{R,j} \tag{LIN.15}
\]
\[
\hat{L}_i^j = s_i^j \hat{L}_{i}^{NR,j} + (1 - s_i^j) \hat{L}_{i}^{R,j} \tag{LIN.16}
\]
\[
\hat{L}_{NR,i}^j = \hat{L}_i^j - \theta_i^j (\hat{R}W_{i}^{NR,j} - \hat{R}W_{i}^{j}) \tag{LIN.17}
\]
\[
\hat{L}_{R,i}^j = \hat{L}_i^j - \theta_i^j (\hat{R}W_{i}^{R,j} - \hat{R}W_{i}^{j}) \tag{LIN.18}
\]

Equations (LIN.15) to (LIN.18) are collinear. Only three of them shall be put in the linearised model.

B.3 Firms

B.3.1 Production

From Section 2.3 under the assumption that firms’ dispersion is a second order phenomenon (i.e. \( \hat{\lambda}_i^j = 0 \)), we have:

\[
\hat{Y}_i^1 = (1 - \alpha) \hat{\xi}_i^1 + (1 - \alpha) \hat{L}_i^1 + \alpha \hat{K}_i^1 - 1 \tag{LIN.19}
\]
\[
\hat{Y}_i^2 = (1 - \alpha) \hat{\xi}_i^2 + (1 - \alpha) \hat{L}_i^2 + \alpha \hat{K}_i^2 - 1 \tag{LIN.20}
\]

where \( \hat{\xi}_i^j = \hat{e}_i^j \).

B.3.2 Capital and labour arbitrage

\[
\hat{r}_i^j + \hat{K}_{i-1}^j = \hat{R}W_i^j + \frac{\hat{p}_i^{\nu,w}}{1 - \hat{p}_i^{\nu,w}} \hat{P}_i^{\nu,w} + \frac{\hat{p}_i^{C}}{1 + \hat{p}_i^{C}} \hat{P}_i^{C} + \hat{L}_i^j \tag{LIN.21}
\]

B.3.3 Dividends

Dividends equal output minus the sum of wages and capital costs, namely:

\[
D_i^j = P_i^j \hat{Y}_i^j - \frac{1}{1 - \alpha} W_i^j (1 + \nu_i^{WR}) L_i^j = P_i^j \hat{Y}_i^j (1 - RM(C_i^j)) \tag{B.1}
\]

Dividing both sides of the equation by \( P_i^j \hat{Y}_i^j Tr_i \):

\[
\frac{D_i^j}{P_i^j \hat{Y}_i^j Tr_i} = \frac{\hat{Y}_i^j}{\hat{Y}_i^j Tr_i} (1 - RM(C_i^j)) \tag{B.2}
\]

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The linearised equation for dividends is:

\[ \hat{d}_t = \hat{Y}_t - (\theta^j - 1)\hat{R}MC^j_t \]

with \(\hat{R}MC^j_t = \hat{R}PC^j_t + (1 - \alpha) \left( \hat{R}W_j^j - \xi^j_t + \frac{\rho^c_j}{1 + \varphi^c_j} \hat{c}^j_t + \frac{\rho^\nu_j}{1 + \varphi^\nu_j} \hat{\nu}^j_t \right) + \alpha \hat{p}_t^j \) (LIN.23)

**B.4 Fiscal Authorities**

Proceeding like for households’ Euler equations, linearising equation (2.70) we get:

\[
0 = (1 - \hat{\beta}_S^R_i) \left( \frac{\hat{\beta}_S^R_i}{1}\left[ \hat{R}_{t+1} - \left( 2\psi S \bar{m}\bar{p}^\nu + \psi S \bar{m}\bar{p}^\sigma \bar{r}^\sigma \right) \bar{p}_t \right] - \hat{R}_t^j - \Delta \hat{C}_{i+1}^j \right) \\
+ (1 - c_i^j) \eta \left( (1 + \hat{\beta}_S^R_i)^2 \Delta \hat{C}_{i+1}^j - \hat{\beta}_S^R_i \Delta \hat{C}_{i+2}^j - \hat{\beta}_S^R_i \Delta \hat{C}_{i}^j \right) \\
+ (1 - c_i^j)(1 - \eta) h_i^j(\hat{\beta}_S^R_i \Delta \hat{C}_{i+1}^j - \Delta \hat{C}_{i}^j) \\
+ (1 - c_i^j)(1 - \eta) \left( \Delta \hat{C}_{i+1}^R_j - \Delta \hat{C}_{i+2}^R_j - \Delta \hat{C}_{i}^R_j \right) \\
- \hat{c}_i^j(1 + \sigma_i^j) \Delta \hat{L}_{i+1}^R_j - \hat{\beta}_S^R_i \Delta \hat{L}_{i+2}^R_j - \hat{\beta}_S^R_i \Delta \hat{L}_{i+2}^R_j \\
- \hat{c}_i^j (1 + \sigma_i^j)(1 - \sigma_i^j) \Delta \hat{L}_{i+1}^R_j - \hat{\beta}_S^R_i \Delta \hat{L}_{i+2}^R_j - \hat{\beta}_S^R_i \Delta \hat{L}_{i+2}^R_j \\
\]

where

\[
\delta \Omega = \frac{1}{1 + \frac{1 - \omega_h^R}{\omega_h^R} \left( \frac{\gamma(1 - \mu)}{1 - \gamma(1 - \mu)} \right) + \sigma_i^j \left( \frac{\gamma(1 - \mu)}{1 - \gamma(1 - \mu)} \right) \frac{(1 + \sigma_i^j \bar{m}\bar{p}^\sigma \bar{r}^\sigma)}{\bar{m}\bar{p}^\sigma \bar{r}^\sigma} - \sigma_i^j} \\
\]

We compare this equation with a budget rule similar to Corsetti et al. (2010):

\[
\hat{G}_t^j = \Psi_x^S \hat{C}_t^j + \frac{\Psi_g^S}{g^y} \hat{y}_t^j + \frac{\Psi_g^d \bar{p}_t^j}{g^y} \bar{p}_t^j 
\]

(LIN.25)
B.5 Financial intermediary

B.5.1 Dividends

Regarding the financial intermediaries, dividends write:

\[
\begin{align*}
\bar{f}_2 (1 + g) \Pi_2 \left( \hat{f}_2 + \hat{T} \right) &= \frac{\theta f_a}{1 + \theta f_a} \left\{ \left( \psi(f_a^1) + \psi(f_a^2) \right) f_a^1 f_a_{t-1}^1 + \left( \psi(f_a^2) + \psi(f_a^1) \right) f_a^2 f_a_{t-1}^1 \\
&\;+ \left( \psi_g(p_a^1) + \psi_e(p_a^1) \right) p_a^1 \hat{p}_a_{t-1}^1 \\
&\;+ \frac{T}{\theta} \left[ \left( \psi(f_a^2) + \psi(f_a^1) \right) f_a^2 f_a_{t-1}^2 + \left( \psi_g(p_a^2) + \psi_e(p_a^2) \right) p_a^2 \hat{p}_a_{t-1}^2 \\
&\;+ \left( \psi(f_a^2) f_a^2 + \psi_g(p_a^2) \right) \hat{T}_{t-1} \right] \right\} \\
\end{align*}
\]

(LIN.26)

\[
\begin{align*}
\bar{f}_2 (1 + g) \Pi_2 \left( \hat{f}_2 + \hat{T} \right) &= \frac{1}{1 + \theta f_a} \left\{ \left( \psi(f_a^2) + \psi(f_a^1) \right) f_a^2 f_a_{t-1}^2 + \left( \psi(f_a^1) + \psi(f_a^2) \right) f_a^1 f_a_{t-1}^1 \\
&\;+ \left( \psi_g(p_a^2) + \psi_e(p_a^2) \right) p_a^2 \hat{p}_a_{t-1}^2 \\
&\;+ \frac{\theta}{T} \left[ \left( \psi(f_a^2) f_a^2 + \psi_g(p_a^2) \right) f_a^2 f_a_{t-1}^1 + \left( \psi_g(p_a^1) + \psi_e(p_a^1) \right) p_a^1 \hat{p}_a_{t-1}^1 \\
&\;+ \left( \psi(f_a^1) f_a^1 + \psi_g(p_a^1) \right) \hat{T}_{t-1} \right] \right\} \\
\end{align*}
\]

(LIN.27)

B.5.2 Zero cash needs condition

We showed that to eliminate a unit root in the model, one should replace one of the budget constraint of governments or Ricardian households by a zero cash needs condition on the financial intermediation market. Once linearised, this condition reads:

\[
0 = f_a^1 f_a_{t}^1 + p_a^1 \hat{p}_a_{t}^1 + \frac{T f_a^2}{\theta} \left( f_a_{t}^2 + \hat{T}_t \right) + \frac{T p_a^2}{\theta} \left( p_a_{t}^2 + \hat{T}_t \right) \\
\]

(LIN.28)
B.6 Monetary Policy, Relative Prices, Inflations

The linearised Taylor rule reads:

\[ \hat{R}_t = \rho \hat{R}_{t-1} + (1 - \rho) \left( \frac{\gamma_1}{4} \sum_{k=1}^{i} \hat{\sigma}_k + r_y \hat{I}_t \right) \]  

(LIN.29)

where the weighted averages for inflation and output in the monetary union are:

\[ \hat{Y}_t^{\text{union}} = \frac{1}{1 + \frac{\gamma_1}{\pi}} \hat{Y}_t^1 + \frac{1}{1 + \frac{\gamma_1}{\pi}} \hat{Y}_t^2 \]  

(LIN.30)

\[ \hat{\sigma}_t^{\text{union,VAT}} = \frac{1}{1 + \frac{\gamma_1}{\pi} \gamma_2} \left( \hat{\sigma}_t^{C,1} + \frac{\gamma}{\gamma_1} (\hat{\sigma}_t^{C,1} - \hat{\sigma}_t^{C,1-1}) \right) \]  

(LIN.31)

By definition of CPI \( i \)

\[ \hat{R}_{PC}^{1, i} = \hat{R}_{PC}^{1} \]

(LIN.32)

\[ \hat{R}_{PC}^{2, i} = -\hat{R}_{PC}^{2} \]  

(LIN.33)

\[ \hat{\sigma}_t^{C,1} = (1 - \alpha^1) \hat{\sigma}_t^1 + \alpha^1 \hat{\sigma}_t^2 \]  

(LIN.34)

\[ \hat{\sigma}_t^{C,2} = (1 - \alpha^2) \hat{\sigma}_t^2 + \alpha^2 \hat{\sigma}_t^1 \]  

(LIN.35)

And the terms of trade depend on inflation differentials between countries:

\[ \hat{T}_t = \hat{T}_{t-1} + \hat{\sigma}_t^1 - \hat{\sigma}_t^2 \]  

(LIN.36)

B.7 Market clearing

From the market-clearing equations, we have:

\[ \hat{Y}_t^1 = \frac{C_1}{Y_1} \hat{C}_{1, t} + \frac{C_1}{Y_1} \hat{C}_{2, t} + \frac{C_1}{Y_1} \hat{C}_{1, t} + \frac{\eta}{Y_1} \hat{I}_{1, t} + \frac{\rho}{Y_1} \hat{I}_{1, t} \]  

(B.4)

\[ \hat{Y}_t^2 = \frac{C_2}{Y_2} \hat{C}_{2, t} + \frac{C_2}{Y_2} \hat{C}_{2, t} + \frac{C_2}{Y_2} \hat{C}_{2, t} + \frac{\eta}{Y_2} \hat{I}_{2, t} + \frac{\rho}{Y_2} \hat{I}_{2, t} \]  

(B.5)
\[ \dot{Y}_1 = (1 - \alpha^1) c_1 c_1^* T T^1 \hat{C}_{1,1}^1 + \alpha^2 c_2 c_2^* T^{1 - \alpha^2} \bar{C}_{1,1}^2 + \gamma y_1 \hat{C}_1^1 + (1 - \alpha^1) i y_1 T a^1 \bar{I}_{1,1}^1 + \alpha^2 i y_2 T^{1 - \alpha^2} \bar{I}_{1,1}^2 \]  
(LIN.37)

\[ \dot{Y}_2 = \alpha^1 c_1 c_1^* T^1 \bar{C}_{2,1}^1 + (1 - \alpha^2) c_2 T^{1 - \alpha^2} \bar{C}_{2,1}^2 + \gamma y_2 \bar{C}_2^1 + \bar{a}^1 i y_1 T a^1 \bar{I}_{2,1}^1 + (1 - \alpha^2) i y_2 T^{1 - \alpha^2} \bar{I}_{2,1}^2 \]  
(LIN.38)

### B.8 Budget constraints

#### B.8.1 Ricardian households

Using the steady state relationships, we linearise as follows:

\[ f_i^1 f_{a^j}^1 = \frac{T_{a^j}}{\bar{\beta}^j (1 + \bar{\gamma})} \left( \frac{\bar{R}}{\bar{R} - \psi(f_{a^j}^1)} \hat{R}_{i-1}^j + \left( 1 - \frac{\psi'(f_{a^j}^1) f_{a^j}^1}{\bar{R} - \psi(f_{a^j}^1)} \right) f_{a^j}^{i-1} - \hat{I}_i^j \right) \]

+ \(1 - s_{i,cd}^j) \frac{1 - \alpha}{1 + \rho_{i,cd}^j} \frac{\theta^j - 1}{\theta^j} \left[ RPC_i^j + \frac{\rho_i^j}{1 + \nu_i^j} \rho_i^{i,j} + RPC_t^j + \hat{R}_i^j \right] 

+ \phi_{i}^j \phi_{i}^{R_i^j} 

+ \left[ RPC_{i}^j + \frac{\rho_i^j}{1 + \nu_i^j} \rho_i^{i,j} + \bar{R}_i^j \right] 

+ \phi_{i}^j \phi_{i}^{R_i^j} 

+ \left[ RPC_{i}^j + \frac{\rho_i^j}{1 + \nu_i^j} \rho_i^{i,j} + \bar{R}_i^j \right] 

+ \phi_{i}^j \phi_{i}^{R_i^j} 

\]  
(LIN.39)

#### B.8.2 Non Ricardian households

Similarly, using the steady state relationships, for the non Ricardian households, we linearise their budget constraint as follows:

\[ \hat{C}_{i}^{NR,j} = \left( 1 - \frac{\phi_{i}^{NR,j}}{(1 + \rho_{i}^{j})RPC_{i}^j cy_i s_{i}^j} \right) \left( RW_{i}^{NR,j} + \hat{I}_{i}^{NR,i} \right) \]

+ \frac{\phi_{i}^{NR,j}}{(1 + \rho_{i}^{j})RPC_{i}^j cy_i s_{i}^j} \left( \phi_{i}^{NR,j} - RPC_{i} - \frac{\hat{v}_{i}^{C,j}}{1 + \rho_{i}^{j} \hat{v}_{i}^{C,j}} \right) \]  
(LIN.40)
B.8.3 Governments

Similarly to non Ricardian households, linearising gives:

\[
\begin{align*}
p_a^t \hat{p}_a^t &= p_a^t \left( \frac{1}{\beta^{t+1}} + \frac{\psi^a(\hat{p}_a^t) \hat{p}_a^t}{\Gamma^a(1 + g)} \right) \left[ \frac{R}{R - \psi^a(\hat{p}_a^t)} \hat{G}_{t-1} + \left( 1 - \frac{\psi^a(\hat{p}_a^t) \hat{p}_a^t}{R - \psi^a(\hat{p}_a^t)} \right) \hat{P}_t + \hat{P}_{t-1} \right] \\
&\quad + \frac{\psi^a(\hat{p}_a^t) \hat{p}_a^t}{\beta^t} \left( \hat{P}_a^t + \hat{P}^t_i \right) + \frac{\psi^a(\hat{p}_a^t) \hat{p}_a^t}{\beta^t} \left( \hat{P}_a^t + \hat{P}^t_i \right) - \hat{g}^t \hat{c}_i^t - \hat{g}^t \hat{h}_i^t \\
&\quad + \frac{\rho^2 \beta^t}{\beta^t} \left( 1 - \frac{\rho^2 \beta^t}{\beta^t} \right) \left( R \hat{W}_i^t + \frac{\rho^2 \beta^t}{\beta^t} \hat{w}_i^t + \hat{L}_i^t + \hat{p}_i^t \right) + \frac{\rho^2 \beta^t}{\beta^t} \left( \hat{C}_i^t + \hat{C}^t_i \right) + \frac{\rho^2 \beta^t}{\beta^t} \right) RMC_i^t \\
&\quad + \frac{\rho^2 \beta^t}{\beta^t} \left( \hat{R}_t^t + \hat{R}^t_i \right) \left( \hat{R}_t^t + \hat{R}^t_i \right) \\
&\quad + \frac{\rho^2 \beta^t}{\beta^t} \left( \hat{G}_t^t + \hat{G}^t_i \right) \left( \hat{G}_t^t + \hat{G}^t_i \right) \\
&\quad + \frac{\rho^2 \beta^t}{\beta^t} \left( \hat{P}_t^t + \hat{P}^t_i \right) \left( \hat{P}_t^t + \hat{P}^t_i \right) \\
&= (\text{LIN.41})
\end{align*}
\]

B.9 Phillips Curves

B.9.1 Prices

From the maximisation of the firm’s programme and the linearisation of the first-order conditions, we have:

\[
0 = \sum_{i=1}^{T} (\beta^t \xi^i)^{T-1} \lambda_{it} \frac{\hat{P}_t^t(\epsilon) \Gamma_{T-1}^i}{\hat{P}_T^t} \left( \frac{\hat{P}_t^t(\epsilon) \Gamma_{T-1}^i}{\hat{P}_T^t} \right)^{-\theta^t} \left( \frac{\hat{P}_t^t(\epsilon) \Gamma_{T-1}^i}{\hat{P}_T^t} \right) \left( \hat{P}_t^t(\epsilon) \Gamma_{T-1}^i - \theta^t \hat{P}_t^t(\epsilon) \Gamma_{T-1}^i - \frac{\theta^t}{\theta^t} \hat{P}_t^t(\epsilon) \Gamma_{T-1}^i \right) \\
0 = \sum_{i=1}^{T} (\beta^t \xi^i)^{T-1} \lambda_{it} \hat{P}_t^t(\epsilon) \Gamma_{T-1}^i - \sum_{k=1}^{T} \left( \hat{P}_t^t(\epsilon) \Gamma_{T-1}^i \right) - \hat{R}MC_{T-1}^i \\
\tag{B.42}
\]

We also know that \( \hat{P}_t^t(\epsilon, t) \) is independent from \( \epsilon \) since all firms solve the same program, and the Calvo process on prices yields:

\[
\frac{\hat{P}_t^t(\epsilon)}{\hat{P}_T^t} = \xi^i \left( \hat{P}_t^t - \gamma^t \hat{P}_{T-1}^t \right). 
\tag{B.43}
\]
We recall equation (2.35), where inflation depends positively on past indexed inflation, future anticipated inflation, relative prices and wages, taxes, total output in country $i$ and negatively on productivity.

**B.9.2 Wages**

We recall equation (2.35):

\[
0 = E_t \sum_{t=1}^{\infty} (\tilde{\beta}^j (1 + g))^T - T_1 \left( \frac{U(C_T(\tau), C_{T-1})}{U(C_T(\tau), C_{T-1})} \right) \nu' \left( \tilde{R}_j, L_{T-1}^i \right) W(G_r^i, G_{r-1}^i) - \nu \left( \tilde{R}_j, L_{T-1}^i \right) W(G_r^i, G_{r-1}^i)
\]

As an example, we derive here the wage Phillips curve for Ricardian households. The first order condition is linearised into:

\[
0 = \sum_{t=1}^{\infty} (\tilde{\beta}_w^j (1 + g)) T - T_1 \left( \frac{R W_i^j + (1 + \theta_w^j (1 + \sigma_j^i)(1 + B^{R,j} - 1))}{\theta_w^j} \right)
\]

\[
+ h_i^j (1 + \sigma_j^i)(1 + B^{R,j}) L_{T-1}^i - ((1 + \sigma_j^i)(1 + B^{R,j} - 1)L_{T-1}^i - \tilde{c}_i^{R,j})
\]

(B.44)

Differentiating between time $t$ and time $t + 1$ yields:

\[
0 = -\tilde{R}_W r_i^j - (1 + \theta_w^j (1 + \sigma_j^i)(1 + B^{R,j} - 1)) \tilde{w}_i^j - h_i^j (1 + \sigma_j^i)(1 + B^{R,j}) L_{t-1}^i
\]

\[
+ ((1 + \sigma_j^i)(1 + B^{R,j} - 1)L_{t-1}^j - \tilde{c}_i^{R,j})
\]

\[
+ \tilde{\beta}_w^j (1 + g)(1 + \theta_w^j (1 + \sigma_j^i)(1 + B^{R,j} - 1))
\]

\[
1 - \tilde{\beta}_w^j (1 + g)
\]

\[
[ - \gamma_w \hat{\Pi}_w^j + \hat{\delta}_w^j - \hat{\delta}_w^j + \tilde{R}_W i_{t+1} - \tilde{R}_W i_t + \hat{\Pi}_w^j + \hat{\Pi}_w^j (\tilde{\eta}_w^j - \tilde{\delta}_w^j)]
\]

(B.45)
And the Calvo process induces:

$$0 = \frac{\bar{c}_W}{\bar{c}_W} \left( \bar{R}_W^{R,i} - \bar{R}_W^{R,i} + (\bar{\gamma}_W^{i} - \gamma_w^{i} \bar{\gamma}_W^{i-1}) - \frac{\bar{v}_C^{i}}{1 + \rho_C^j} (\bar{\gamma}_W^{i} - \gamma_w^{i} \bar{\gamma}_W^{i-1}) \right) + (1 - \bar{c}_W) \partial w^{R,j}_i$$  \hfill (B.46)

$$\delta w^{R,j}_i = \frac{\bar{c}_W}{1 - \bar{c}_W} \left( \bar{R}_W^{R,j} - \bar{R}_W^{R,j} + (\bar{\gamma}_W^{i} - \gamma_w^{i} \bar{\gamma}_W^{i-1}) + \frac{\bar{v}_C^{i}}{1 + \rho_C^j} (\bar{\gamma}_W^{i} - \gamma_w^{i} \bar{\gamma}_W^{i-1}) \right).$$  \hfill (B.47)

where one may read \( \bar{R}_W^{R,i} - \bar{R}_W^{R,i} + (\bar{\gamma}_W^{i} - \gamma_w^{i} \bar{\gamma}_W^{i-1}) \) more simply as nominal wage inflation of the Ricardians in deviation from its steady state. In the end, the wage Phillips curve for the Ricardian households writes:

$$\bar{R}_W^{R,i} - \bar{R}_W^{R,i} + (\bar{\gamma}_W^{i} - \gamma_w^{i} \bar{\gamma}_W^{i-1}) + \frac{\bar{v}_C^{i}}{1 + \rho_C^j} (\bar{\gamma}_W^{i} - \gamma_w^{i} \bar{\gamma}_W^{i-1}) =$$

$$\beta^i (1 + g) \left( \bar{R}_W^{R,j} - \bar{R}_W^{R,j} + \frac{\bar{v}_C^{i}}{1 + \rho_C^j} (\bar{\gamma}_W^{i} - \gamma_w^{i} \bar{\gamma}_W^{i-1}) \right)$$

$$+ \frac{(1 - \bar{c}_W) (1 + g) (1 - c_w^{i})}{\bar{c}_W (\theta_w^{i} (1 + c_w^{i}) + B^{R,j}) (1 - c_w^{i})} \left[ -\bar{R}_W^{R,i} - \bar{R}_W^{R,i} + (1 + c_w^{i}) (1 + B^{R,j}) (\hat{L}_i^{R,i} - \hat{L}_i^{R,i} + \hat{C}_i^{R,i}) \right].$$

(LIN.43)

The wage Phillips curve for the non Ricardian households writes:

$$\bar{R}_W^{NR,i} - \bar{R}_W^{NR,i} + (\bar{\gamma}_W^{i} - \gamma_w^{i} \bar{\gamma}_W^{i-1}) + \frac{\bar{v}_C^{i}}{1 + \rho_C^j} (\bar{\gamma}_W^{i} - \gamma_w^{i} \bar{\gamma}_W^{i-1}) =$$

$$\beta^i (1 + g) \left( \bar{R}_W^{NR,j} - \bar{R}_W^{NR,j} + \frac{\bar{v}_C^{i}}{1 + \rho_C^j} (\bar{\gamma}_W^{i} - \gamma_w^{i} \bar{\gamma}_W^{i-1}) \right)$$

$$+ \frac{(1 - \bar{c}_W) (1 + g) (1 - c_w^{i})}{\bar{c}_W (\theta_w^{i} (1 + c_w^{i}) + B^{NR,j}) (1 - c_w^{i})} \left[ -\bar{R}_W^{NR,i} - \bar{R}_W^{NR,i} + (1 + c_w^{i}) (1 + B^{NR,j}) (\hat{L}_i^{NR,i} - \hat{L}_i^{NR,i} + \hat{C}_i^{NR,i}) \right].$$

(LIN.44)

with \( B^{NR,j} = \frac{\rho}{\sigma} \frac{\theta-1}{\theta} \frac{(1-\eta)(1-c_w^{i})}{\rho(1-\eta)(1+c_w^{i})+\rho^{\eta/\sigma}} \) so that the slope of the wage Phillips Curve very marginally depends on the household’s type.

In all, the level of inflation on wages hinges positively on future anticipated inflation, past inflation, taxes. The larger the discount factor \( \beta^i \), the more sensitive is wage inflation to inflation expectations. The larger the probability to adjust prices \( 1 - \xi^{i} \) (i.e. the more flexible are prices and wages), the less inflation depends on expectations. Inflation also positively depends on labour demand and consumption: households demand a higher wage to work more and consequently consume more.
C Monetary policy in a currency union

C.1 Defining aggregate indexes for the union

In the absence of exact derivations for both the union aggregate price index and the union total output gap, we turn to the definitions of such indexes. Namely, the aggregate price index comes from:

\[(1 + \nu^C_{union}) CPI^C_{union} = (1 + \nu^C_{1}) CPI^C_{1} + (1 + \nu^C_{2}) CPI^C_{2}\]  
(C.1)

therefore the VAT-included CPI inflation for the union rewrites

\[\Pi_{union,VAT} = \frac{(1 + \nu^C_{union}) CPI^C_{union}}{(1 + \nu^C_{union-1}) CPI^C_{union-1}} \Pi_{union,1,VAT} + \frac{C^C_{union} C^C_{1}}{C^C_{1-1}} \Pi_{union,2,VAT}\]

(C.2)

Our approximation relies on the choice of weights on inflations taken at their steady state values. As a result, we obtain:

\[\Pi_{union,VAT} = \frac{1}{1 + (1 + \nu^C_{2}) CPI^C_{2}} \Pi_{C,1,VAT}^{C,1,VAT} + \frac{1}{1 + (1 + \nu^C_{1}) CPI^C_{1}} \Pi_{C,2,VAT}^{C,2,VAT}\]

(C.3)

Similarly, we define the aggregate union production as follows:

\[p_{union} = p^1_t \gamma^1_t + p^2_t \gamma^2_t\]  
(C.4)

therefore, taking the static weights at their steady state value, the union total output gap rewrites:

\[\gamma_{union} = \frac{\gamma^1_{union}}{\gamma_{C,union}} = \frac{p^1_t}{p^1_{union}} \frac{1}{1 + \frac{\theta}{\gamma^1_t}} \gamma^1_t + \frac{p^2_t}{p^2_{union}} \frac{1}{1 + \frac{\theta}{\gamma^2_t}} \gamma^2_t = \frac{1}{1 + \frac{\theta}{\gamma^1_t}} \gamma^1_t + \frac{1}{1 + \frac{\theta}{\gamma^2_t}} \gamma^2_t\]  
(C.5)

C.2 Should inflation be net of VAT?

The ECB measures inflation with the Harmonised Index of Consumer Prices (HICP) which includes VAT. However, changes in the tax rate seldom happen and one may expect the ECB not to react to such
shocks. It is then possible to define a VAT-excluded alternative:

$$\Pi_{i_{\text{union}}} = \frac{1}{1 + \frac{\text{CPI}_{i}^{\text{CPI-1}}}{\text{CPI}_{i}}} \Pi_{i}^{CPI,1} + \frac{1}{1 + \frac{\text{CPI}_{i}^{\text{CPI-1}}}{\text{CPI}_{i}}} \Pi_{i}^{CPI,2}$$  \hspace{1cm} (C.6)$$

as the inflation targeted by the Central Banker. One can also easily consider other Taylor rules, with quarter to quarter inflation, output growth, GDP or GDP growth...

### C.3 Defining GDP in this context

When there are no taxes on products or subsidies on production, $Y_{i}^{t}$ is equal to GDP. *Stricto sensu* $Y_{i}^{t}$ is actually the total value added produced in country $i$. In a model with taxes, in particular VAT, $Y_{i}^{t}$ actually differs from GDP. In nominal GDP is:

$$GDP_{i_{\text{nominal}}}^{\text{nominal}} = p_{i}^{t}Y_{i}^{t} + v_{i}^{CPI}CPI_{i}^{t}(C_{i}^{t} + I_{i}^{t})$$  \hspace{1cm} (C.7)$$

Changes in the VAT rate are considered as price effects so that the growth index of GDP in real terms can be measured by:25

$$Index_{i_{\text{GDP}}}^{\text{GDP}} = \frac{p_{i}^{t-1}Y_{i}^{t} + v_{i}^{CPI}CPI_{i}^{t-1}(C_{i}^{t} + I_{i}^{t})}{p_{i}^{t-1}Y_{i}^{t-1} + v_{i}^{CPI}CPI_{i}^{t-1}(C_{i}^{t-1} + I_{i}^{t-1})}$$  \hspace{1cm} (C.8)$$

An additional issue is the production of financial intermediation services. In the present model we have considered that they are not located in country 1 or 2. Still union wide GDP should incorporate this production.

$$GDP_{i_{\text{union,nominal}}}^{\text{nominal}} = \sum_{i \in \{1,2\}} p_{i}^{t}Y_{i}^{t} + v_{i}^{CPI}CPI_{i}^{t}(C_{i}^{t} + I_{i}^{t}) + \sum_{i \in \{1,2\}} \psi \left( FA_{i-1}^{t-2} \frac{FA_{i-1}^{t-1}}{p_{i-1}^{t-1}Y_{i}^{t-1}Tr_{i-1}} \right) FA_{i-1}^{t-1} + \psi \left( \frac{PA_{i-1}^{t}}{p_{i-1}^{t}Y_{i}^{t}Tr_{i-1}} \right) PA_{i}^{t-1}$$  \hspace{1cm} (C.9)$$

For financial production, the volume index is the growth rate of financial assets in real terms, the corresponding growth index in volume is:

$$Index_{i_{\text{GDP}}_{\text{vol}}}^{\text{GDP}_{\text{vol}}} = \frac{\sum_{i \in \{1,2\}} p_{i}^{t-1}Y_{i}^{t} + v_{i}^{CPI}CPI_{i}^{t-1}(C_{i}^{t} + I_{i}^{t}) + \sum_{i \in \{1,2\}} \psi \left( FA_{i-1}^{t-2} \frac{FA_{i-1}^{t-1}}{p_{i-1}^{t-1}Y_{i}^{t-1}Tr_{i-1}} \right) FA_{i-1}^{t-1} + \psi \left( \frac{PA_{i-1}^{t}}{p_{i-1}^{t}Y_{i}^{t}Tr_{i-1}} \right) PA_{i}^{t-1}}{\sum_{i \in \{1,2\}} p_{i}^{t-1}Y_{i}^{t-1} + v_{i}^{CPI}CPI_{i}^{t-1}(C_{i}^{t-1} + I_{i}^{t-1}) + \sum_{i \in \{1,2\}} \psi \left( FA_{i-1}^{t-2} \frac{FA_{i-1}^{t-1}}{p_{i-1}^{t-1}Y_{i}^{t-1}Tr_{i-1}} \right) FA_{i-1}^{t} + \psi \left( \frac{PA_{i-1}^{t}}{p_{i-1}^{t}Y_{i}^{t}Tr_{i-1}} \right) PA_{i}^{t-2}}$$  \hspace{1cm} (C.10)$$

---

25This formula corresponds to the growth rate of real GDP when measured in over the quarter overlap chained linked volumes. This technique is seldom used by national accountants but other techniques require to isolate quarters of the same year which is not possible in the present model.
D Firms and households dispersion

Contrary to a flexible price model, price and wage nominal rigidities create distributional issues. As firms and agents cannot reset their prices and wages every period, some agents will keep a constant price (in the absence of indexation) for non-zero periods of time. As a result, at each date, a full distribution of prices and wages coexists.

Working with aggregate variables, this distributional issue rarely matters and is usually invisible. It does only matter for aggregation across firms and households. In particular, the link between an individual household consumption/labour and the aggregate consumption/labour determines the aggregate Euler equations 2.25, investment decision 2.26, Tobin’s Q 2.27 and wage Phillips curves 2.35.

In this appendix, we show that under the reasonable assumption that the variance of dispersion in consumption, wages, prices or labour is small, the first order conditions aggregates exactly at the first order. As an alternative in the literature, papers such as Erceg, Henderson, and Levin (2000) resort to the assumption of complete contingent claims markets for consumption and labour. In this case, households can perfectly ensure against consumption and labour fluctuations, and consumption is constant across similar households.

D.1 Labour dispersion

Recall the aggregate production (Equation 2.58) that depends on the index of labour dispersion across firms. This index is defined as follows:

\[
\left(\Delta_i\right)^{\theta_i-1} = \frac{1}{p^i P} \int_0^{p^i P} \left( \frac{L_i^i(\epsilon, t)}{L_i^i / p^i P} \right)^{\theta_i-1} d\epsilon \tag{D.1}
\]
Using a second-order Taylor expansion of $\mathcal{H}$, we obtain:

$$
\begin{align*}
\mathcal{H} &= 1 + \frac{1}{p^i} \int_0^{p^i} \theta^i - 1 \left[ \frac{L^i(\epsilon, t)}{L^i_0/p^i} - 1 \right] - \frac{1}{2\theta^i} \left[ \frac{L^i(\epsilon, t)}{L^i_0/p^i} - 1 \right]^2 d\epsilon \\
&= 1 - \frac{1}{2p^i} \int_0^{p^i} \left[ \frac{L^i(\epsilon, t)}{L^i_0/p^i} - 1 \right]^2 d\epsilon \\
\text{as } L^i &= \int_0^{p^i} L^i(\epsilon, t) d\epsilon 
\end{align*}
$$

We notice that the first order term disappears in the Taylor expansion of the dispersion index. As a result, it appears that labour dispersion is a purely second order phenomenon under the reasonable assumption that $\frac{L^i(\epsilon, t)}{L^i_0/p^i} - 1$ remains small. As we study a first-order linearised model, this assumption of a small variance of dispersion seems rather reasonable. We can therefore neglect labour dispersion for production aggregation.

**D.2 Ricardian first order conditions**

Similarly, it is possible to aggregate the Ricardians’ first order conditions at the first order, assuming that individual consumption and labour fluctuations around the aggregate level are of small magnitude.

In the Euler equation, the investment decision and Tobin’s Q equation for the Ricardian agents, the part of the subjective discount factor depending on $\tau$ is given by (eliminating external habits constant across $\tau$):

$$
\begin{align*}
 f \left( C^i_{t+1}(\tau), C^i(\tau), l^i_{t+1}(\tau), l^i_t(\tau) \right) &= \frac{U'(C^i_{t+1}(\tau), C^i(\tau)) V(l^i_{t+1}(\tau), L^i_t)}{U'(C^i_{t}(\tau), C^i_{t-1}) V(l^i_{t}(\tau), L^i_{t-1})} \\
&= \left( \frac{C^i_{t+1}(\tau)}{C^i_t(\tau)} \right)^{(1-\sigma_i)(1-\eta)-1} \left( \frac{l^i_{t+1}(\tau)}{l^i_t(\tau)} \right)^{\eta_i} \left( \frac{1 - \kappa(1 - \sigma_i) \left[l^i_{t+1}(\tau)(L^i_t) - h^i_{t+1}(\tau) \right]^1 + \sigma_i}{1 - \kappa(1 - \sigma_i) \left[l^i_t(L^i_{t-1}) - h^i_t \right]^1 + \sigma_i} \right)^{-1} 
\end{align*}
$$

(D.3)
Conducting a Taylor expansion of this function \( f \) around the mean \[
\begin{pmatrix}
\frac{C_{i+1}^{R,j}}{(1-\mu)(1-\mu)\mu N}, & \frac{C_{i}^{R,j}}{(1-\mu)(1-\mu)\mu N}, & \frac{L_{i+1}^{R,j}}{(1-\mu)(1-\mu)\mu N}, & \frac{L_{i}^{R,j}}{(1-\mu)(1-\mu)\mu N}
\end{pmatrix}
\] and at the second order gives:

\[
f \left( C_{i+1}(\tau), C_{i}(\tau), L_{i+1}(\tau), L_{i}(\tau) \right) = f \left( \frac{C_{i+1}^{R,j}}{(1-\mu)^3\mu N}, \frac{C_{i}^{R,j}}{(1-\mu)^3\mu N}, \frac{L_{i+1}^{R,j}}{(1-\mu)^3\mu N}, \frac{L_{i}^{R,j}}{(1-\mu)^3\mu N} \right)
\]

Aggregate subjective discount factor

\[
\begin{align*}
&1 + \left( (1-\sigma_{c}^{i}) (1-\eta^{i}) - 1 \right) \left( \frac{C_{i+1}^{R,j}(1-\mu^{i})n^{i}N}{C_{i+1}^{R,j}} - 1 \right) \\
&- \left( (1-\sigma_{c}^{i}) (1-\eta^{i}) - 1 \right) \left( \frac{C_{i}^{R,j}(1-\mu^{i})n^{i}N}{C_{i}^{R,j}} - 1 \right) \\
&+ \sigma_{c}^{i} \left( 1 - \frac{\sigma_{c}^{i} - 1}{\sigma_{c}^{i}} \right) \frac{\kappa (1 - \sigma_{c}^{i})}{1 - \kappa (1 - \sigma_{c}^{i})} \left( \frac{L_{i+1}^{R,j}(L_{i+1}^{R,j} - h_{i})^{1+\sigma_{c}^{i}}}{L_{i+1}^{R,j}} - 1 \right) \\
&- \sigma_{c}^{i} \left( 1 - \frac{\sigma_{c}^{i} - 1}{\sigma_{c}^{i}} \right) \frac{\kappa (1 - \sigma_{c}^{i})}{1 - \kappa (1 - \sigma_{c}^{i})} \left( \frac{L_{i}^{R,j}(L_{i}^{R,j} - h_{i})^{1+\sigma_{c}^{i}}}{L_{i}^{R,j}} - 1 \right) \\
&+ o \left( \left\| \left( \frac{C_{i}^{R,j}(1-\mu^{i})n^{i}N}{C_{i}^{R,j}} - 1 \right), \left( \frac{L_{i}^{R,j}(1-\mu^{i})n^{i}N}{L_{i}^{R,j}} - 1 \right) \right\|^{2} \right)
\end{align*}
\]

(D.4)

As \( \int_{R} C(\tau, t)d\tau = C_{i}^{R,j} \) and \( \int_{R} L(\tau, t)d\tau = L_{i}^{R,j} \), first order terms disappears when we aggregate the subjective discount factor (or directly the equation) by integrating over all \( \tau \). In the end, the first order conditions for the households is valid at the first order as long as consumption and labour dispersion are indeed second order phenomenons, that is their respective variance is small.

The same methodology can be used to justify the use of aggregate Lagrange multipliers in both the price and wage Calvo setting programs.

## E Determination of a unique and stable steady state

### E.1 Debt elastic spreads

Schmitt-Grohé and Uribe (2003) show on the case of a small open economy that a debt elastic interest rate premium pins down a unique steady state debt level because this specification modifies the steady state of households’ Euler equation. Following the general specification used for monetary union models, we introduce such a modification in our model for the same result: equation (STEADY.1) relates the
discount factor and the steady state real monetary rate with the steady state asset to GDP ratio. Put differently, with this specification the steady state is unique in $\bar{a}$ and verifies equation (STEADY.1).

Moreover, the Euler equations on private consumption shows that if the spread is an increasing function of the asset to GDP ratio, when assets are below their steady state value, consumption will decrease today and yield an increase in assets converging to their steady state (consumption could also increase tomorrow, a strategy which would eventually violate the no Ponzi condition).

A symmetric result holds for public assets ($\bar{p}a$) with our maximizing government. Otherwise a budget rule must incorporate a feedback loop between the level of public debt and public spending (or another fiscal instrument).

Because the adjustment is through final demand, in terms of country i net financial position, it will show on the trade balance rather than the net transfers to the rest of the union.

Putting together the budget constraints in each country, aggregating Ricardian and non Ricardian households, yields:

$$FA_i + PA_i = (R_{i-1})(FA_{i-1} + PA_{i-1}) + TB_i + FD_i - \text{Spreads}_i \quad (E.1)$$

If there were no fees for financial intermediation ($FD_i = \text{Spreads}_i = 0$), any temporary imbalance of the trade balance ($TB_i$) would make country i’s net financial assets ($FA_i + PA_i$) diverge.

Convergence is nevertheless not obtain through financial transfers with the rest of the union but because imbalances of the trade balance are not simply transitory. The spread paid on assets in the Euler equation implies that as long as the net financial position is not at steady state, either on the private or public side, private or public consumption will compensate and the trade balance will not return to its steady state. This is the main mechanism for convergence to the steady state.

With the introduction of financial intermediation, when the net financial position deviates from its steady state value, fees paid to the financial market increase in both countries and so do the dividends paid to domestic owners of financial firms. Thus, this mechanism is in first approach neutral on the convergence to the steady state.
E.2 An inconvenient eigenvalue

However, to close the model, the four steady state net financial positions can not be chosen freely. Moreover, the model does not converge towards its steady state when the four asset dynamics are written.

From the model, we have the following six budget constraints:

\[
P_{t-1}^A = \begin{pmatrix} R_{t-1} - \psi \left( \frac{PA_{t-1}^j}{P_{t-1}^2 Y^T r_{t-1}} \right) \end{pmatrix} \begin{pmatrix} PA_{t-1}^j - P_{t-1}^2 C_{t-1}^j - \Phi_{t-1}^j \end{pmatrix}
\]

(E.2)

\[
FA_{t-1}^j = \begin{pmatrix} R_{t-1} - \psi \left( \frac{FA_{t-1}^j}{P_{t-1}^j Y^T r_{t-1}} \right) \end{pmatrix} \begin{pmatrix} FA_{t-1}^j - CPI_{t}^j(1 + v_{t}^{p_{t}^j})C_{t}^{R_{t-1}} - CPI_{t}(1 + v_{t}^{p_{t}^j})I_{t}^{j} \end{pmatrix}
\]

(E.3)

\[
W_{t}^{NR_{t}} L_{t}^{NR_{t}} + \Phi_{t}^{NR_{t}} = CPI_{t}(1 + v_{t}^{p_{t}^j})C_{t}^{NR_{t}}
\]

(E.4)

Adding up these equations union-wide yields:

\[
FA_{t}^1 + FA_{t}^2 + PA_{t}^1 + PA_{t}^2 =
\]

\[
R_{t-1}(FA_{t-1}^1 + FA_{t-1}^2 + PA_{t-1}^1 + PA_{t-1}^2) + W_{t}^{1} L_{t}^{1} + (1 - v_{t}^{K_{t}})CPI_{t}^{1} r_{t}^{K_{t}} K_{t-1}^{1} + (1 - v_{t}^{D_{t}^1})D_{t}^{1} + (1 - v_{t}^{D_{t}^2})D_{t}^{2} + (1 - v_{t}^{F_{t}^D} )FD_{t}^{1} + \Phi_{t}^{1} + W_{t}^{2} L_{t}^{2} + (1 - v_{t}^{K_{t}})CPI_{t}^{2} r_{t}^{K_{t}} K_{t-1}^{2} + (1 - v_{t}^{D_{t}^2})D_{t}^{2} + (1 - v_{t}^{D_{t}^1})FD_{t}^{2} + \Phi_{t}^{2} - CPI_{t}^{1}(1 + v_{t}^{p_{t}^1})(C_{t}^{1} + I_{t}^{1}) - CPI_{t}^{2}(1 + v_{t}^{p_{t}^2})(C_{t}^{2} + I_{t}^{2}) + v_{t}^{D_{t}^1} W_{t}^{1} L_{t}^{1} + v_{t}^{1} CPI_{t}^{1}(C_{t}^{1} + I_{t}^{1}) + v_{t}^{D_{t}^2} W_{t}^{2} L_{t}^{2} + v_{t}^{2} CPI_{t}^{2}(C_{t}^{2} + I_{t}^{2}) + v_{t}^{F_{t}^D} W_{t}^{1} L_{t}^{1} + v_{t}^{1} CPI_{t}^{1}(C_{t}^{1} + I_{t}^{1}) + v_{t}^{F_{t}^D} W_{t}^{2} L_{t}^{2} + v_{t}^{2} CPI_{t}^{2}(C_{t}^{2} + I_{t}^{2}) + (1 - v_{t}^{D_{t}^1})D_{t}^{1} + (1 - v_{t}^{D_{t}^2})D_{t}^{2} + (1 - v_{t}^{F_{t}^D} )FD_{t}^{1} + (1 - v_{t}^{D_{t}^1})D_{t}^{1} + (1 - v_{t}^{D_{t}^2})D_{t}^{2} + (1 - v_{t}^{F_{t}^D} )FD_{t}^{1} - P_{t}^{1} G_{t}^{1} - \Phi_{t}^{1} - P_{t}^{2} G_{t}^{2} - \Phi_{t}^{2} - \psi \left( \frac{FA_{t-1}^{1}}{P_{t-1}^{2} Y^T r_{t-1}} \right) FA_{t-1}^{1} - \psi \left( \frac{FA_{t-1}^{2}}{P_{t-1}^{2} Y^T r_{t-1}} \right) FA_{t-1}^{2} - \psi \left( \frac{PA_{t-1}^{1}}{P_{t-1}^{2} Y^T r_{t-1}} \right) PA_{t-1}^{1} - \psi \left( \frac{PA_{t-1}^{2}}{P_{t-1}^{2} Y^T r_{t-1}} \right) PA_{t-1}^{2}
\]

(E.5)
Rearranged, after simplification of taxes, transfers into:

\[
FA_1^t + FA_2^t + PA_1^t + PA_2^t = R_{t-1}(FA_{1-1}^t + FA_{2-1}^t + PA_{1-1}^t + PA_{2-1}^t) \\
+ W_1^t (1 + \nu_1^w) L_1^t + CPI_1^t r_1^k 1^t + D_1^t + W_2^t (1 + \nu_2^w) L_2^t + CPI_2^t r_2^k 2^t + D_2^t \\
- CPI_1^t (C_1^t + I_1^t) - CPI_2^t (C_2^t + I_2^t) - P_1^t G_1^t - P_2^t G_2^t + FD_1^t + FD_2^t \\
- \psi \left( \frac{FA_{1-1}^t}{P_{1-1}^t Y_{1}^t Tr_{1-1}} \right) FA_{1-1}^t - \psi \left( \frac{FA_{1-1}^t}{P_{2-1}^t Y_{2}^t Tr_{1-1}} \right) FA_{1-1}^t \\
- \psi^g \left( \frac{PA_{1-1}^t}{P_{1-1}^t Y_{1}^t Tr_{1-1}} \right) PA_{1-1}^t - \psi^g \left( \frac{PA_{1-1}^t}{P_{2-1}^t Y_{2}^t Tr_{1-1}} \right) PA_{1-1}^t
\]  

(E.6)

We have assumed that the labour, capital and goods markets are in equilibrium, hence we can cancel out the difference between two classic break downs of production (i.e. total revenue minus total demand in both countries) and simplify fisims and financial dividends (union wide financial production), the sum yields:

\[
CN_t = R_{t-1} CN_{t-1}.
\]  

(E.7)

Assuming the nullity of either initial or final conditions of total net asset holding in the monetary union, i.e. \(CN_0 = 0\) or \(CN_\infty = 0\) is sufficient to have financial intermediaries clearing their position vis-à-vis the central bank at each period.

All the asset dynamics but one and the zero cash needs constraint (2.79) at steady state is enough to make the model Walrasian while verifying the fourth budget constraint. On the contrary, resorting to the four asset dynamic equations would introduce a diverging dynamic in the model (verifying \(CN_t = R_{t-1} CN_{t-1}\)).  

The zero cash needs constraint is thus a necessary condition with a reasonable economic interpretation.

### F Other IRFs

\[A property we verified numerically.\]
y-axis in p.p. deviation from steady state

Figure 10: IRFs to a one percent productivity shock (autocorrelated)
y- axis in p.p. deviation from steady state

Figure 11: IRFs to a one percent preference shock (autocorrelated)
Figure 12: IRFs to a 100 basis points monetary policy shock

y-axis in p.p. deviation from steady state
Figure 13: IRFs to a one percent government spending shock (autocorrelated)
Figure 14: IRFs to a one percent government spending shock (autocorrelated)

y-axis in p.p. deviation from steady state
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<tr>
<td>$\tilde{w}_i^j(\tau), \tilde{W}_i^{NL}, \tilde{W}_i^{NR}$</td>
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<td>$fa_i = FA_i/r_i^\tau r_i^\tau, \psi(fa_i)$</td>
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<td>$PA_i^i$</td>
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<td>$CPI_i^j = CPI_i/p_i^j$</td>
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Table 3: Definition of the endogenous variables
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<tr>
<td>$n$, $N$, $n^i$</td>
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<td>$g$</td>
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<tr>
<td>$\theta^i$, $\theta^w_i$</td>
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<tr>
<td>$\alpha^i$</td>
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<td>$\beta^i$, $\beta^S_i$</td>
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<td>$\sigma^c_i$, $\sigma^l_i$</td>
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<tr>
<td>$\eta^i$</td>
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<td>$h^c_i$, $h^l_i$</td>
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<td>$h^g_i$</td>
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<td>$\kappa^i$</td>
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<tr>
<td>$\gamma^i_p$, $\gamma^i_w$</td>
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<tr>
<td>$1 - \xi^i$, $1 - \xi^w_i$</td>
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<tr>
<td>$\mu^i$</td>
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<tr>
<td>$\delta$</td>
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<td>$\alpha$</td>
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<tr>
<td>$\theta^i$, $\theta^{fa}_i$, $\theta^c$</td>
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<th><strong>Policy parameters</strong></th>
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<td>$\bar{\nu}_c$, $\bar{\nu}_w$</td>
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<tr>
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<td>$\phi_f^i$, $\phi^{R;i}_f$, $\phi^{NR;i}$</td>
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<td>$\Pi^i$, $\rho$, $r_T$, $r_y$</td>
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<thead>
<tr>
<th><strong>Endogenous parameters</strong></th>
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<tr>
<td>$\bar{R}$</td>
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<tr>
<td>$cy^i$, $gy^i$, $iy^i$</td>
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<td>$s^c_i$, $s^l_i$</td>
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Table 4: Definition of the parameters
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